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An Experimental Investigation of the Hole-drilling Technique for Measuring Residual Stresses in Welded Fabricated Steel Tubes

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Title: An Experimental Investigation of the Hole-drilling Technique for Measuring Residual Stresses in Welded Fabricated Steel Tubes.

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Among semi-destructive methods of measuring residual stresses in elastic materials, the blind hole-drilling strain-gage method is one of the best because it is simple, economical and accurate. It is based on the measurement of strains disturbed by machining a small diameter shallow hole in the test piece. The strains measured in three known directions permit the determination of the direction and magnitude of principal stresses and subsequently of any stress in any direction.

This thesis presents the investigation of residual stresses in the longitudinal direction of a welded fabricated steel tube of 22 inch diameter, relating to a series of holes drilled in one half of a

circular section of the tube. An initial assumption, substantiated later, was the existence of a uniform field of residual stresses through the thickness of the tube. Several methods for determining calibration coefficients are documented. The values of longitudinal stresses once computed are presented in a smooth curve. A straight line approximation is recommended for use in further studies of the effects of residual stresses on failure loads.

AN EXPERIMENTAL INVESTIGATION OF THE HOLE-DRILLING TECHNIQUE FOR MEASURING RESIDUAL STRESSES IN WELDED FABRICATED STEEL TUBES

by

CHAU MONG TRAN

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IN LOVING MEMORY OF MY PARENTS,
OF MY PARENTS-IN-LAW, AND OF
MY FOSTER FATHER.

TO MY FOSTER MOTHER,

TO MY BROTHER,

TO MY WIFE,

TO MY CHILDREN,

TO MY CHILDREN-IN-LAW.

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4A	Data reduction coefficient or calibration coefficient
4B	Data reduction coefficient or calibration coefficient (for minimum principal stress)
D	Difference of strains, $\varepsilon_a - \varepsilon_c$
d	Outside diameter of the steel tube
d_1	Inside diameter of the steel tube
E	Modulus of elasticity
I	Moment of inertia
M_y	Bending moment at first yield
P	Axial load
P_y	Axial load causing complete yielding of the cross-section
R	Distance of the gage to hole center
R_o	Radius of the drilled hole
r	R/R_o, ratio of R to R_o
S	Sum of strains, $\varepsilon_a + \varepsilon_c$
s	Section modulus
t	Thickness
α	Angle of longitudinal stress to principal stress σ_1
β	Angle of direction (gage) a to principal stress σ_1
ε_a	Strain measured by gage a
ε_b	Strain measured by gage b
ε_c	Strain measured by gage c
ε_L	Longitudinal strain
ε_T	Transverse strain
σ_1	Principal stress, maximum

σ_2	Principal stress, minimum
σ_L	Longitudinal residual stress
σ_T	Circumferential or transverse residual stress
σ_y	Yield stress

CHAPTER I

INTRODUCTION

1.1 REVIEW OF LITERATURE

In many occurrences, one of the predominant factors contributing to structural failures and fatigue in welded parts, pipes, rolled structural shapes and finished structures, is the residual stresses which existed in the part before being put into service. These stresses are introduced either by the initial imperfections in fabrication or during the manufacturing processes due to the operations of casting, molding, laminating, heat treating, cold or hot rolling, machining or welding. In other instances, residual stresses can appear due to installation procedures or the dead weight of the structure. In general, residual stresses tend to reduce the strength in stability, fatigue and fracture (7, 13, 14); in some situations, however, their existence might improve the strength (13, 14). Consequently, knowledge of the distributions and magnitude of the residual stresses is a necessary prelude to any analytical investigation of the effects of these stresses on the behavior of the structure.

The directions and magnitudes of these stresses can be determined by several methods:

- Methods termed "destructive" consist of removing metal by machining, slicing (5, 7) grinding or etching and measuring elastic strains of the remaining section.

In 1888, Lalakoutsky (7) reported on a "sectioning method" to determine longitudinal stresses in steel bars by slicing longitudinal strips from the bar and measuring their change in length. The other methods are based on similar principles. That is the internal stresses are relieved or changed when the specimen is reduced to strips or pieces of smaller cross-section. The test piece submitted to the experiment of one of these methods, is unusable in its original configuration and thus the method is termed destructive.

Non-destructive or x-ray methods are based on the principle of modification of the texture of metal grains revealed by x-rays. The results are correlated to the texture of the same metal under known stress conditions. Internal stresses present in the metal piece are finally determined.

Semi-destructive methods consist of machining a shallow hole in a test piece by means of a rotating cutter or an abrasive jet, and of measuring the strains disturbed in three directions by a strain-gage rosette: the stresses present in the test piece, for any direction around the hole, are computed from these strains.

Because of the complexity of their use, the slicing techniques have become less acceptable.

Using x-ray does not lend itself to field conditions and stresses can be detected only in the surface layer to a depth of one thousandth of an inch.

The two hole-drilling methods attempt to minimize the disadvantages of the previous discussed methods.

The "Abrasive Jet Machining" method consists of directing a controlled stream of gas containing fine abrasive particles against a workpiece to chip away tiny particles of material making a small hole.

The strains relieved are measured by means of a strain-gage and internal stresses relaxed around the drilled hole can be calculated. However, because the "Abrasive Jet Machining" method (2) requires costly equipment- the hole-drilling or hole-relaxation method, consisting of drilling a hole in the test piece by means of a rotating cutter and of measuring the strains relieved with a strain-gage, was used in this study. Both are termed "semi-destructive", because the amount of metal removed is small. The part itself once drilled for the disturbed strains can be repaired by means of a rivet or a plug and recovers the integrity of its strength and properties.

The drilling method was first conceived by G. Sachs (6) who, in 1927, worked with specially shaped pieces (e.g., those with round or rectangular cross-sections). In 1932, Josef Mathar in Germany (1,3) developed the technique and measured the deformations around the drilled hole by means of a mechanical extensometer. His apparatus has been subject to criticism, because the vibrations during drilling cause the strain readings to be unsteady and irregular. The use of the method developed by Mathar is restricted to cases where the stresses are considered to be uniform through the thickness.

In 1948, Soete in Belgium (1) improved the technique and extended its use to cases of non-uniform stress field.

During the evolution of the hole-drilling technique the strains relieved were at first measured by an extensometer (1) then by some photoelastic materials bonded to the specimen surface or by a brittle lacquer sprayed on its surface (2,10). Finally, in the present time, bonded resistance wire strain gages constitute the most widely used process of strain measurement (2,10,12). The combined method - hole drilling and strain gage - finds its use in most work, because of its simplicity, ease of strain measurement and stress calculation and its higher precision. In fact, it has been proven in a specific job that the hole-drilling method needs only 10 man hours, where as the layer removal method requires about 70 to 80 man hours (1), the former saves at least 85 per cent of labor and time.

In spite of its numerous advantages, the hole-drilling method shows two relatively unfavorable conditions:

a) Difficulty of obtaining a perfect alignment of the drilled hole with respect to the strain gages (problem of centering the drill bit in a gage rosette).

b) Spot residual stresses introduced during drilling.

The first disadvantage can be avoided by using a commercial fixture: a precision milling guide capable of centering the hole to the center of the rosette within ±0.001 inch.

A method of determining the effect of the spot residual stresses caused by drilling, described in detail in ~~paragraph~~ section 2.3.1 overcomes the second shortcoming. The spot residual stresses introduced during drilling, although measured at the same time by the strain gage, are separated from the stresses of interest by the use of calibration coefficients.

1.2 OBJECTIVE OF THIS INVESTIGATION

The purpose of this investigation is to study longitudinal residual stresses present in a welded fabricated steel tube, to determine their magnitudes and distributions along a semi-circular cross-section and to check the correctness of measurements by a test of statical equilibrium of the stress distribution. The tube chosen was 5/16 inch thick, 22 inches in diameter and 6 feet long.

The following outlines the two major steps in determining residual stresses:

1) Drilling holes on the outside and inside surfaces, in one half of a circular section of the tube, two feet from the edge, then measuring the strains by means of strain gage rosettes, calculating the corresponding longitudinal stresses using the calibration coefficients determined in Step 2, and plotting these stresses in a curve.

2) Performing experiments of calibration. The first one is conducted according to a calibration process of strain-relaxation due to drilling: a plate specimen is submitted to a known applied tension, then a hole is drilled in the plate. The strain due to the applied stress and the strain induced by the drilling are measured. This calibration yields the first pair of calibration coefficients.

The second experiment is carried out according to a calibration process of separating strains and consequently the induced stresses and applied stresses. This experiment gives the second pair of calibration constants.

In the calibration study, a steel plate was used having the same general characteristics as the steel tube.

1.3 ORGANIZATION OF THE REPORT

The following is a documentation of longitudinal residual stresses present in a welded fabricated steel tube.

The hole-drilling method was used in the experiment of drilling outside and inside of a semi-circular section of a full steel tube. Calibration coefficients were determined through experiments of pre-loading a steel plate specimen.

A procedure is presented which separates stresses introduced by the drilling process.

The results are shown as a stress distribution around the circumference of the tube. An independent check of translational and rotational equilibrium is presented. A stress distribution is recommended for use in a study of the effect of residual stresses on failure loads of fabricated steel tubes.

CHAPTER II

ANALYTICAL APPROACH

2.1 GENERAL CONSIDERATIONS

As the investigation is carried out on a welded cylindrical steel tube, it is appropriate here to describe the process by which welded fabricated steel tubes are commonly made. Usually, before welding, several cycles of cold-rolling of a flat plate are repeated until the two opposite edges come together to form a cylinder or "can". A can is then completed by welding along the longitudinal seam. The length of the can is usually limited to 10 feet by this manufacturing process, but several cans can be welded together end-to-end to yield the desired length.

A possibility of longitudinal weld tearing in a finished member when loaded, is avoided by staggering the weld between cylinders making the weld in one can 180 degrees out-of-phase to the weld in the next can.

This manufacturing process generally introduces:

a) Initial residual stresses from the flat plate before forming the "can" (5,14).

b) Circumferential residual stresses by repeated cold-rolling (5).

c) Longitudinal residual stresses due to longitudinal welding (5)(Shrinkage during cooling).

d) A bending moment on the cross-section of the wall of the

(3) because it is possible that the edges of the rolled plate do not meet exactly, thus requiring some "pulling together" of the edges prior to or during the welding operation (incomplete "spring back")(5).

Among these stresses, measurement of longitudinal residual stresses is the main objective of this analysis.

2.2 HOLE-DRILLING METHOD

2.2.1 Presentation of the method. The measurement of residual stresses cannot be performed by conventional surface techniques such as photo-elastic coatings or strain gages, because the measuring device ignores the past story of the parts. It measures strain after it is bonded to the part.

For this investigation, the hole-drilling method was chosen for use in determining the "locked-in" stresses. The stress field being disturbed by drilling, stresses are relaxed around the hole and relieved strains are measured.

2.2.2 Terminology of stresses and mathematical expressions. When a hole of smaller diameter ($d = 2R_o$) is drilled in a part subjected to residual stresses, a strain relaxation occurs. If there is only one stress σ_1, the strains relieved at a point P from a distance R of the center A of the hole are called radial strain ε_r and tangential strain ε_t (10), in Figure 2.1.

$$\varepsilon_r = -\sigma_1(1+\mu)/2E(1/r^2-3\cos 2\alpha/r^4 + 4\cos 2\alpha/((1+\mu)r^2)) \quad (1)$$

$$\varepsilon_r = -\sigma_1(1+\mu)/2E(-1/r^2+3\cos 2\alpha/r^4 - 4\cos 2\alpha/((1+\mu)r^2))$$

t

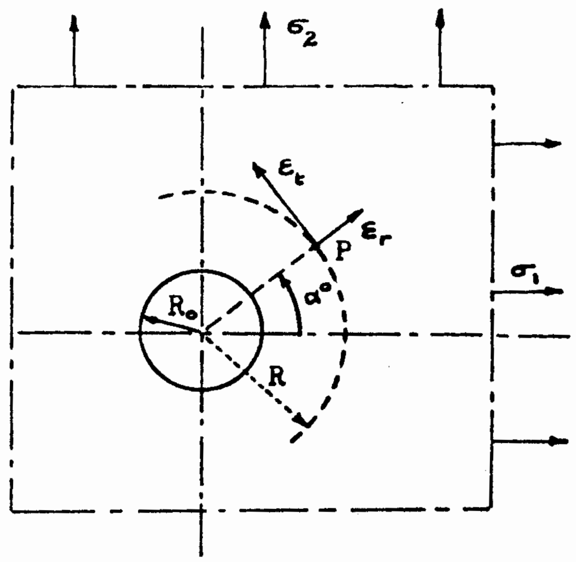


Figure 2.1 Radial and tangential strains

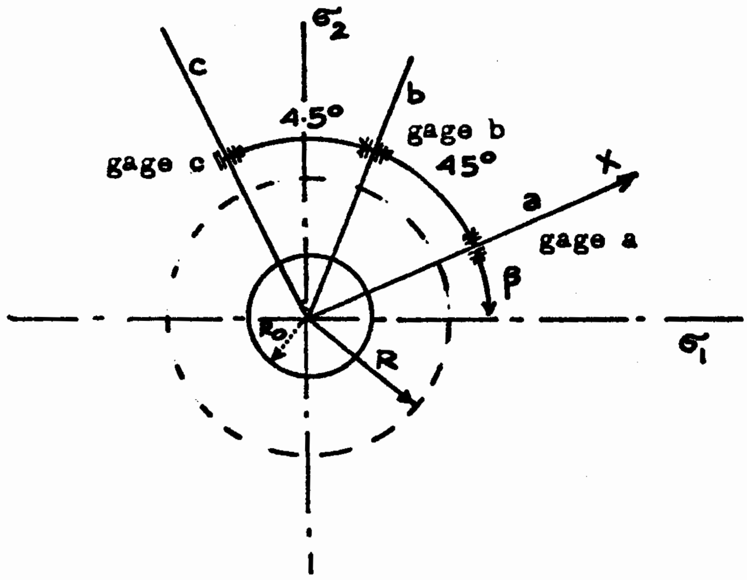


Figure 2.2 Position of gages with respect to principal stress σ_1.

where α = angle of radial strain to direction of stress σ_1.

$r = R/R_o$

μ = Poisson's ratio.

The equation (1) is used by Redner (10)

E = Modulus of elasticity.

The above expressions show that the radial and tangential strains vary sinusiodally along a circle of radius R and can be expressed as:

$$\varepsilon_r = (A + B \cos 2\alpha)\sigma_1$$

$$\varepsilon_t = (C + D \cos 2\alpha)\sigma_1$$

If both stresses σ_1 and σ_2 are present at the same time (σ_1 and σ_2 orthogonal to each other), the expression becomes:

$$\varepsilon_r = (A + B \cos 2\alpha)\sigma_1 + (A + B \cos 2(\alpha+90))\sigma_2 \tag{2}$$

Only the radial strain expression is retained here because each gage of the rosette 125RE used in the main experiment is unidirectional and measure the radial strain for each of the directions a,b,c.

Coefficients A and B in expression (2) can be calculated from equations (1) for any given material and at any specified radius of measruements. They can also be determined experimentally.

2.2.3 Measuring strains using the strain gage rosette. In order to measure residual stresses σ_1 and σ_2 and their direction β (see Figure 2.2) to a selected reference X, three strain measurements are required. These three measurements will provide three equations from which the three unknown, σ_1 and σ_2 stresses and β direction, can be calculated.

As shown in Figure 2.2, the strain gage rosette comprises three separate gages in directions a,b,c spaced 45 degrees apart, on a radius R. The strains can be established from equation (2) by letting the angles:

$$\alpha_a = \beta \qquad \alpha_b = \beta \qquad \alpha_c = \beta + 90°$$

Solving for stresses, one obtains (Derivation of equations (3) is given in Appendix I):

$$\sigma_1 = ((A+B\cos 2\beta)\varepsilon_a - (A-B\cos 2\beta)\varepsilon_c)/4AB\cos 2\beta$$

$$\sigma_2 = ((A+B\cos 2\beta)\varepsilon_c - (A-B\cos 2\beta)\varepsilon_a)/4AB\cos 2\beta \qquad (3)$$

$$\operatorname{Tan} 2\beta = (\varepsilon_a - 2\varepsilon_b + \varepsilon_c)/(\varepsilon_a - \varepsilon_c)$$

The equations are for a hole drilled in a macroscopically homogeneous, isotropic material subjected to a biaxial stress.

Equations (3) provide good results when gages a and c are approximately positioned along principal stresses (10). But if the directions are grossly misjudged and gages a and c give the largest spread, the following equations (4) provide better results (Derivation of equations (4) given in Appendix II):

$$\sigma_1 = \frac{(A + B\sin 2\beta)\varepsilon_a - (A - B\cos 2\beta)\varepsilon_b}{2AB(\sin 2\beta + \cos 2\beta)}$$

$$\sigma_2 = \frac{(A + B\cos 2\beta)\varepsilon_b - (A - B\sin 2\beta)\varepsilon_a}{2AB(\sin 2\beta + \cos 2\beta)} \qquad (4)$$

Equations (3) can be rewritten in the following form to simplify the numerical calculations (10)(Derivation of equations (3a) is given in Appendix III)

$$\sigma_1 = S/4\ A + D/4\ B \cos 2\beta$$

$$\sigma_2 = S/4\ A - D/4\ B \cos 2\beta \qquad (3a)$$

with $\quad \text{Tan}\ 2\beta = (S - 2\varepsilon_b)/D$

where $\quad S = \varepsilon_a + \varepsilon_c$

$\qquad\qquad D = \varepsilon_a - \varepsilon_c$

The coefficients 4A and 4B can be computed from equation (1) or obtained from curves in Figure 2.3; these curves established by S. Redner (10) yield one pair of values of 4A and 4B for one value of $r = R/R_o$. In this study, they were also determined experimentally.

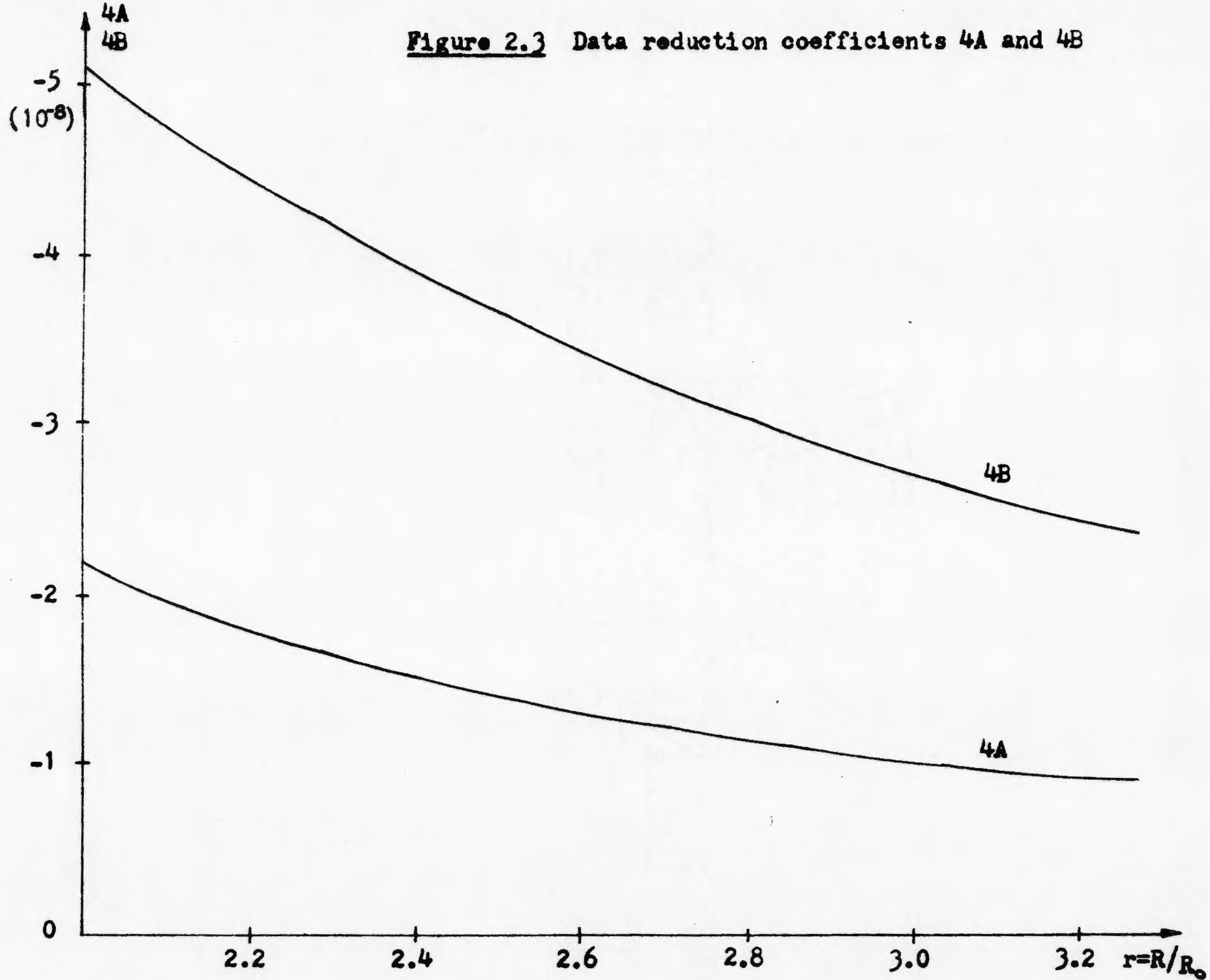
Each system of equations (3), (3a) and (4) can be used to compute the magnitudes and directions of the principal stresses. However, for the case of this investigation, the system of equations (3a) is most often applied because of their simplicity.

A computer program using equations (3a) to calculate the directions and magnitudes of the principal stresses, when the strains ε_a, ε_b and ε_c are known, is given in Appendix IV.

Generally the directions of the principal residual stresses calculated do not coincide with the longitudinal and circumferential directions of the tube. A graphical method using a stress Mohr's circle (11) permits the orientation of the longitudinal and circumferential residual stresses.

When the strains in the directions of gages a, b and c are known, a graphical solution using a strain Mohr's circle (11) allows the calculation of the longitudinal and circumferential strains

Figure 2.3 Data reduction coefficients 4A and 4B



(see Appendix VII).

A series of computer program presented in section 2.4 constitutes a second method for solving the above mentioned problems (see Appendices VI and VIII).

2.2.4 Limit of strains for $D/D_o = 1.0$. As mentioned previously, the hole-drilling method is based on the fact that drilling a hole in a stress field disturbs the equilibrium of the stresses, thus causing measurable strains at the surface of the part, adjacent to the hole. Previous investigators have found that the deformations of the surface in a thick test piece, approach a limiting value. This is because the stresses relieved from the removal of the metal, at some distance below the surface will have no effects on the deformations at the surface. It has been found that this limiting value occurs at a hole depth varying from one to two times the hole diameter (1). Mathar (6) found this limit to be 1.5 to 2, while Bush, Kromer (2) and Redner (10) suggested 1.0.

In this investigation, it was found that little change in the strains occurs after a hole depth to diameter ratio of 1.0 is reached. For convenience, it will be shown in Chapter III.

2.2.5 Variation in residual stress field. An assumption has been made that the residual stresses existing in the tube are uniform through its thickness. This assumption will be numerically substantiated in section 3.2.1.b.

At this time, the following observation about the distribution of residual stress is noted.

According to S. Redner (10), the criterion of a uniform stress field is:

> When the residual stresses are uniform in depth, the strains in the immediate vicinity of the hole are fully relieved (the curve reaches an asymptotical value) when the depth equals approximately one diameter of the hole.

In this study, it was found that strains relieved were small for the first layers of material, but increased with depth. When the hole depth reached the value of hole diameter, strains attained their maximum and when plotted remained parallel to the horizontal axis.

The quasi-stationary values of the stresses computed for several depth values close to the constant diameter value will be shown in Chapter III a substantiation of the above assumption.

2.3 DETERMINATION OF CALIBRATION COEFFICIENTS BY EXPERIMENTAL METHOD

The hole-drilling or hole-relaxation method of measuring residual stresses is based on the measurement of strains distributed by machining a small hole in the test piece. A theoretical solution for the strain at any point on the surface of a drilled hole is not known and furthermore the relaxed strain is not uniform across the surface of the part. A strain gage at best can measure the average strain across the area it covers. Because of these limitations, most investigators believe that a calibration procedure is necessary. Calibration operations consist of applying known stresses to a test piece and measuring corresponding strains. These known stresses and strains permit the establishment of calibration coefficients which can be used in similar experiments to calculate stresses when the strains relieved are known. In this investigation, results are presented using calibration coefficients obtained by: a) theoretical calculations, b) published literature, c) experimental work performed as part of this research.

2.3.1 Determination of calibration coefficients by the method of strain relaxation due to drilling. This method consists of computing the strain relaxed when a hole is drilled in a steel plate subjected to a known state of stress. In this method no account is made for stresses induced by drilling.

The test set up is simple. A steel plate having a shape shown in Figure 3.18 is installed in a testing system capable of applying a tension force.

The following presents the major steps in this method:

1) A known tension (for example a tension giving a stress $\sigma_1 = 0.50\ F_y$) is applied to the plate and strain ε_a is measured by a strain gage rosette B; ε_a is the strain due to the known force. Plot point a in Figure 2.4. (Note: no hole has been drilled as yet.)

2) The tension is released. The strain reading comes back to zero. A hole is drilled to a depth equal to the hole diameter. There is another strain reading, but it is reset to zero. The same tension is reapplied to the plate and a new strain ε_b is read and recorded; ε_b is the total strain due to the application of the force and the existence of the drilled hole. Plot point b (strain ε_b, stress σ_1) in Figure 2.4.

3) The strain-relaxation $\Delta\varepsilon$ is the difference between the two measured strains ε_a and ε_b.

As all strain readings have been zeroed before and after drilling, the influence of any unknown residual stresses in the test plate is eliminated. Their effect has been eliminated by using the experimental procedure outlined above. All readings were zeroed before the force was applied. As the known force is applied before and after drilling, only the effects of the applied load and any stresses induced by drilling are recorded.

This calibration method yields a pair of calibration coefficients 4A and 4B, their computation will be shown in Chapter III.

2.3.2 Determination of calibration coefficients by the method of strain-separation. The second method of determining the calibration coefficients separates the unknown stresses induced during the drilling process.

It has been found that, during the drilling operation, significant stresses are introduced in a general specimen by drill bits or rotating cutters. According to Bush and Kromer (2), their magnitude can be of the order of ±10000 psi. Now the change in strain introduced by drilling a hole in any test piece when the latter is under load, is a function of initial residual stresses, spot stresses induced by drilling and applied stresses. Therefore, for an accurate calibration, it is necessary to isolate the values of existing unknown residual stresses in our test plate from stresses induced by drilling and from those caused by our applied loads.

The test set-up is similar to the previous method but there are some differences in making readings and computing strains. The same test plate is installed in a testing system capable of applying a tension force. The experimental procedure is as follows:

1) Before drilling, zero strain indicator. Apply a known load (σ_1 = 0.40 F_y) to the specimen. Record the strain ε_a from the strain indicator. Plot point a of Figure 2.5.

2) Maintaining the stress σ_1. A hole is drilled to the desired depth. Record the corresponding strain ε_b. Plot in the same graph point b (ε_b,σ_1).

3) Reduce the magnitude of stress to σ_2 (σ_2 = 0.10 Fy) and record the corresponding strain ε_d. Plot point d(ε_d,σ_2). Locate point c, which is the intersection of line oa and a line through d parallel to the strain axis (see Figure 2.5).

4) Extend line bd cutting the strain axis at point e (graphical interpolation). ε_e is the strain with no load on the plate. As there is no load applied, the segment oe represents the strain due to initial residual stresses and spot stresses introduced by the drilling operation.

5) Referring to ordinate σ_1, the segment ab represents the difference of two strains $\varepsilon_a - \varepsilon_b = \Delta\varepsilon_T$ which is the strain-relaxation due to drilling under applied stress σ_1. $\Delta\varepsilon_T$ is the sum of two strains: 1) strains due to drilling effects and unknown residual stresses existing in the plate and 2) strains due to applied stress. From 0, a parallel line to eb is drawn cutting ab at b'. The segment bb' parallel and equal to eo represents the strain due to drilling effects and unknown residual stresses in the plate ($\Delta\varepsilon_R$). The segment b'a represents the strain due to the applied stress σ_1($\Delta\varepsilon_1$).

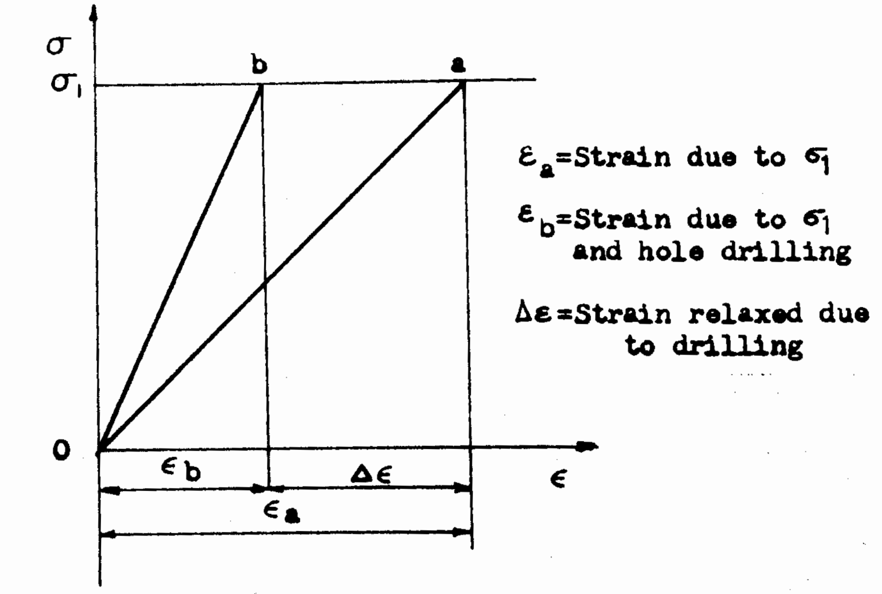


Figure 2.4 Strain-relaxation due to drilling

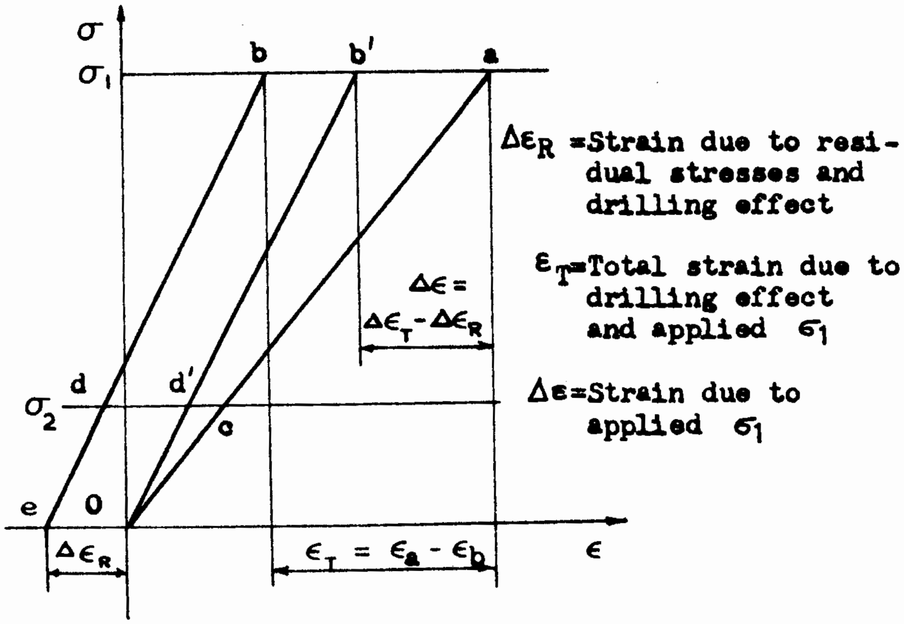


Figure 2.5 Strain separation

Finally $\Delta\varepsilon_1$ is the strain caused by the applied stress σ_1.

6) At ordinate σ_2, dd' parallel and equal to eo, represents the strain due to drilling effects and unknown residual stresses, and the segment d'c is the strain due to the applied load σ_2.

7) The above strain- relaxation has been obtained from a known applied stress for a determined direction. In the case of the gage rosette, it is necessary to calculate the strain-relaxation for three different directions (a,b,c, 45 degrees apart) under an uniaxial load.

2.3.3 Experiment of tensioning a steel plate specimen for calibration. The purpose of this experiment is to determine the modulus of elasticity of the steel of the plate, to check the existence of initial residual stresses and stresses induced by drilling and finally, to determine the calibration coefficients, first by the method of strain-relaxation due to drilling, then by the method of strain separation.

Experimental set-up and results are described in detail and illustrated with numerical values in Chapter III, sections 3.1.3 and 3.2.2.

2.4 COMPUTER PROGRAMS

In this part, three computer programs will be presented pertaining to 1) the calculations of principal stresses, 2) of longitudinal and circumferential stresses and 3) of longitudinal strain, when radial strains in three directions (gages) a,b,c are known. Two more computer programs will be discussed relating to force and moment balance before and after calibration.

As the system of the actual longitudinal residual stresses is in equilibrium in the tube, the summation of forces and moments with respect to an axis must equal zero. The first program computes the balance of forces and moments of measured longitudinal residual stresses. The second program deals with the balance pertaining to residual stresses recalculated using calibration coefficients.

2.4.1 Computer program for calculating principal stresses σ_1 and σ_2 when strains in directions a,b,c are known. Calculations are based on equations (3a):

$$\sigma_1 = S/4A + D/4B \cos 2\beta$$
$$\sigma_2 = S/4A - D/4B \cos 2\beta \qquad (3a)$$

$$\text{Tan } 2\beta = (S - 2\varepsilon_b)/D$$

with

$$S = \varepsilon_a + \varepsilon_c$$

$$d = \varepsilon_a - \varepsilon_c$$

These equations are used by Redner (10).

The computer program is shown in Appendix IV.

2.4.2 Computer program for calculating longitudinal and circumferential stresses when strains in three directions a,b,c are known. In this program, the equation of a Mohr's circle constructed with principal stresses is established. The equation of the straight line representing both longitudinal and circumferential directions is also computed. The abscissas of the points of intersection between the line and the circle will be the longitudinal and circumferential stresses. (see Figure A.6.1).

The computer program is presented in Appendix VI.

2.4.3 Computer program for calculating longitudinal and circumferential strains when strains in three directions are known. In this program, the equation of a strain Mohr's circle is established. Also computed is the equation of the line of the longitudinal strain direction which is the same for the circumferential strain direction. (The angle between longitudinal and circumferential direction is 90 degrees for the tube; but this angle is double and equals 180 degrees in Mohr's circle.) The abscissas of the points of intersection between the strain Mohr's circle and the line, will be the longitudinal and circumferential strains. (See Figure A.7.1)

The computer program is presented in Appendix VIII.

CHAPTER III

EXPERIMENTAL PROGRAM

This chapter documents the testing procedure and presents the numerical results of the laboratory tests.

Laboratory tests consist of experiments of drilling outside and inside holes in the steel tube and of an experiment of tensioning a steel plate specimen for calibration.

3.1 PHYSICAL TESTS

The first experiments consist of drilling a series of eleven holes on the outside surface of a welded fabricated steel tube.

3.1.1 Drilling holes on outside surface.

A) Material and gages.

1) The steel tube specimen used has the following characteristics:

Length: 6 feet

Outside diameter: 22 inches

Thickness of wall: 5/16 inch

Section modulus: 118.79 cubic inches

Yield axial load: 870.8 kips

Bending moment at first yield M_y: 4859 in. kip

A-36 American made plate

The steel used in the tensioning experiment has the following specifications:

Conform to ASTM-36.75

Yield strength: 40.90 ksi

Ultimate strength: 61.50 ksi

Ultimate strength: 61.50 ksi

Modulus of elasticity: 29 000 ksi

Chemical analysis:	Carbon:	.14%
	Manganese:	.67%
	Phosphor:	.009%
	Sulfur:	.018%
	Silicium:	.22%

2) Strain-gages used are of two types:

a) Rosette 0.125" (see Figure 3.2)

Gage type: EA .06-125 RE-120

Resistance in ohms: 120.0 ± 0.2%

Lot number: R - A 35 AD 48

Quantity: 5 gages per box

Option: S

Gage factor at 75°F: Nominal 2.03 + 1.0%

Manufactured by Micro-Measurement M-M, Romulus, Michigan

This rosette is used with a drill bit of 0.125 inch to drill holes of 0.125 inch diameter.

b) Rosette 0.062"

Gage type: EA .06-062 RE - 120

This resistance strain gage has specifications similar to those of Rosette 0.125 in. except for its smaller dimensions and is used for holes 0.062 in. in diameter.

Figure 3.1 Steel tube

Figure 3.2 Strain gage rosette

B) Apparatus.

1) Milling guide.

As mentioned previously, the difficulty of obtaining a good alignment of the bored hole with respect to the gages can be overcome by the use of a precision milling guide (see Figure 3.3).

The apparatus used in this experiment is a RS-200 special milling guide (12) capable of allowing an alignment within ±0.001 inch of the rosette center and insuring the concentricity and guidance of the milling bar.

This guide is supported by a rigid body adjustable in two orthogonal directions. The body is easily attachable to curved or flat surfaces by means of an adjustable swivel tripod. When the gage is cemented to the test piece, alignment is realized by putting the microscope into the guide and centering the guide over the center of the gage rosette by acting adjusting screws. When alignment is done, the microscope is removed and the milling bar armed with rotating cutter is introduced into the guide. The cutter currently used is of 0.125 in. diameter.

According to Vigness and Rendler (4), end mills are normally equipped with cutting edges on the end and side of the mill. The mill will cut either with an axial or lateral feed motion. But, in 1966, Greenweld and Rendler (10) made a complete and valuable study on this matter and found an optimum shape for the drilling tool, insuring that the boring progresses in a straight line, without side pressure or friction at the non-cutting edge. This shape of drill bit has been applied to the equipment used in the present experiment and allowed a smooth drilling without side pressure or side friction.

The milling bar is also equipped with a special micrometer screw adapter to allow drilling in small increments in depth (generally 0.01" per increment). After drilling, the diameter of the hole is measured by the micrometer inside the microscope. This measured diameter is necessary to compute the ratio $r = R/R_o$ distance of the gage to the center, to radius of the hole, to determine the data reduction coefficients 4A and 4B leading to the computations of the principal stresses.

2) Strain indicator.

After the installation of the rosette, its three gages are soldered to six wires (one pair of wires to one gage) connected to the strain indicator (see Figures 3.4 and 3.5). The gage factor is set up to the required value (2.03 for the EA .06-125 RE - 120 Rosette) and when the reading is reset to zero the test should yield a reading of ±3 400 microstrains for each gage connected to the strain indicator.

C) Experimental set up

The experimental set up comprises the following operations:

1) Location of holes, marking.

The longitudinal residual stresses were assumed to be symmetric with respect to the dimetral plan passing through the longitudinal weld. Therefore only the investigation of the stresses on a semi-circular section of the tube was considered. A section two feet from the edge (see Figure 3.6) has been defined and the holes were drilled on a semi-circular section commencing from the weld (see Figure 3.7).

Figure 3.3 Milling guide

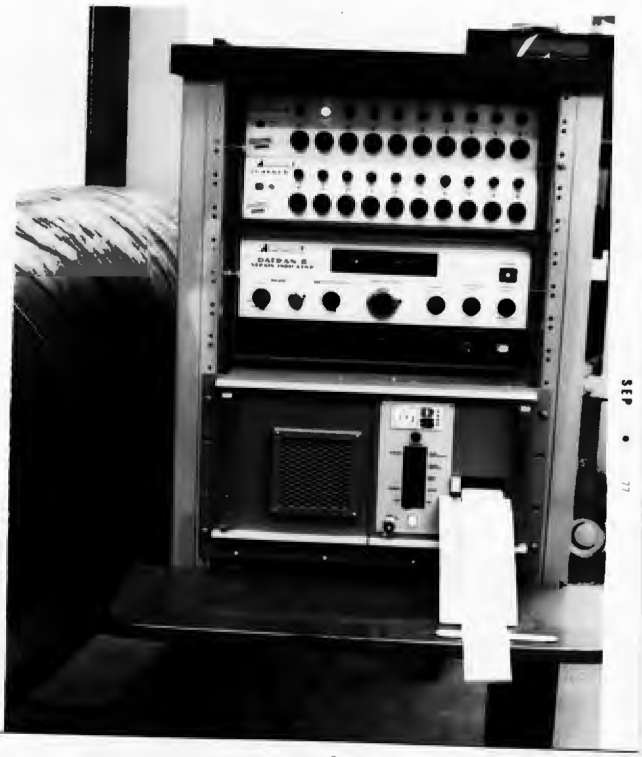


Figure 3.4 Strain indicator

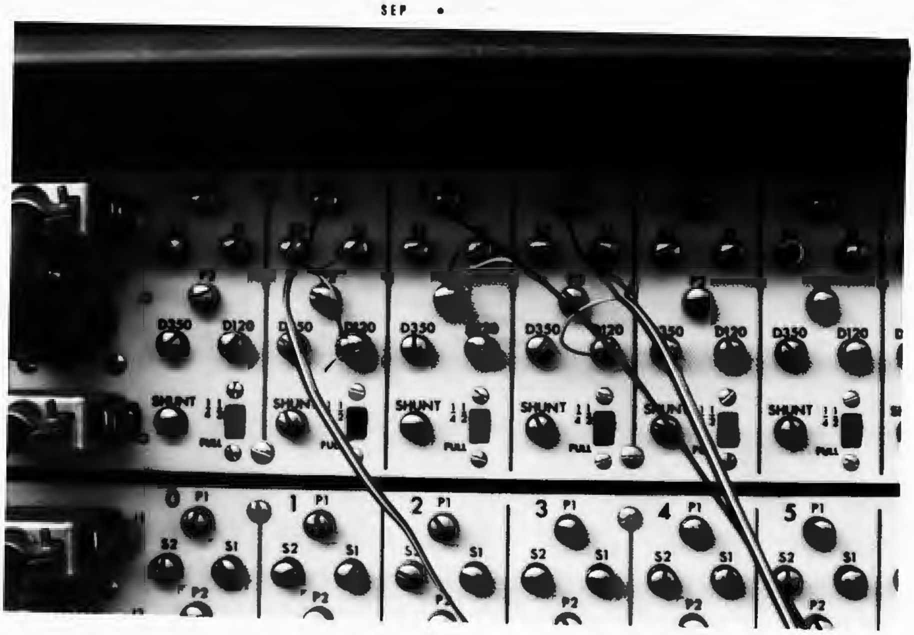


Figure 3.5 Strain indicator (rear view)

Eleven holes were drilled and located from the weld as follows:

Hole	Angle from weld
1	45°
2	90°
3	135°
4	180°
5	22°30
6	67°30
7	112°30
8	157°30
9	near weld
A	11°15
B	33°45

Each hole is marked with a sharp steel blade on the wall of the pipe, strongly enough to leave a faint sign of a cross even after sanding, but not so strongly as to create extraneous residual stresses in the tube.

2) Cleaning the area ready for cementing the gages and the swivel feet for mounting the milling guide.

-Sanding. As the outside wall of the steel tube is covered with scales caused by the hot-rolling operation, it is required to clean with sand cloth, around the marking sign, an area large enough to receive the gage and the tripod. Sanding has to be done cautiously to prevent introduction of induced stresses. Sanding is stopped when the area assigned to receive the cementing of the gage and the swivel feet are polished and clean. A good sanding requires about

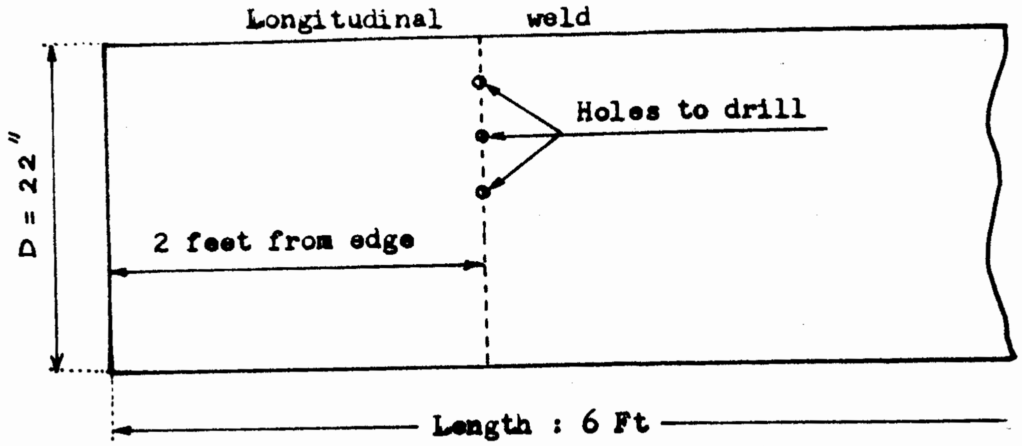


Figure 3.6 Location of hole section

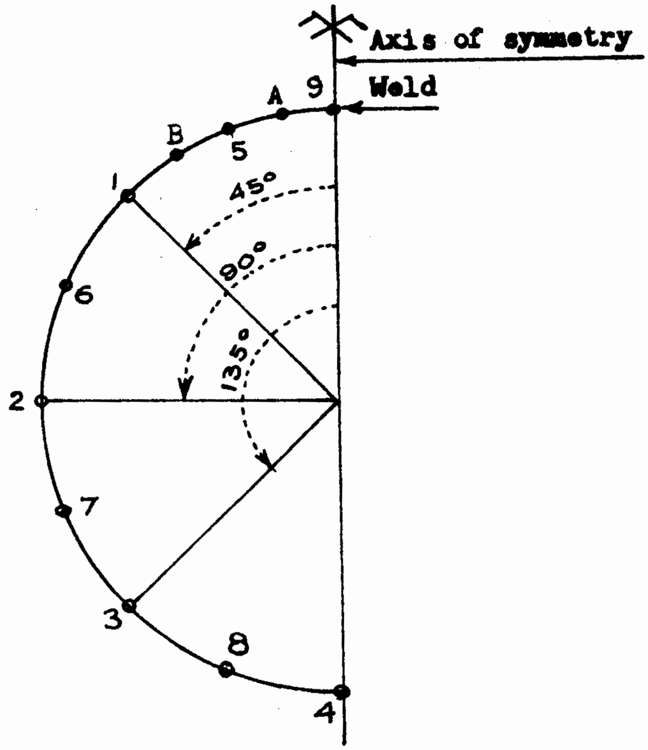


Figure 3.7 Position of drilled holes

one hour of work for one set-up.

-Chemical cleaning. After sanding is completed, a solution of acetone is applied three to four times to remove dust or dirt. Then it is cleaned with "Conditioner A", a water-based acidic surface cleaner. Finally, a water-based alkaline surface cleaner "Neutralizer 5" is applied to neutralize the effects of the remaining acidic solution and to complete the cleaning operation.

3) Cementing gage.

125 RE type rosette is shown in Figure 3.8 with its three directions a,b,c. It is more convenient to adopt a unique orientation of the gage rosette with respect to the longitudinal axis to make it uniform for all set-ups. The adopted orientation is shown in Figure 3.9.

The gage rosette is first put on a flat surface and two short scotch tape bands are mounted on it. The mount is set in place for a gage positioning check. A thin coat of "200 Catalyst" is applied on the back side of the rosette. Then a coat of "M-Bond 200 Adhesive" is painted on the film side of "200 Catalyst". Now it is required to spread with a thumb the scotch tape bands, sticking the gage on the steel tube and squeezing out the excessive adhesive.

4) Soldering gage

Two terminals of each gage of the rosette are soldered to a pair of wires connecting to the rear side of the strain indicator (see Figure 3.5). To check soldering and cementing of the gage, a load is applied by hand at the surrounding areas of each gage. The gages should record a slight change on the indicator.

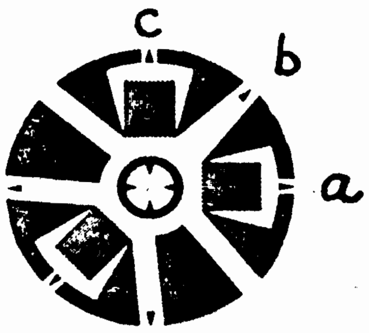


Figure 3.8 Rosette pattern enlarged

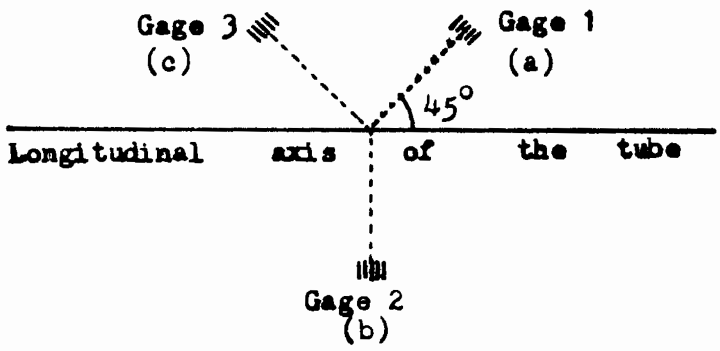


Figure 3.9 Orientation of rosette gages

5) Cementing tripod.

For mounting the milling guide, the three adjustable swivel feet must be cemented in suitable positions (see Figure 3.10).

A "Grip-Cement" powder is mixed with a "Grip-Cement" liquid to form a resinous type dental cement suited for attaching the swivel feet to the test piece.

6) Strain indicator set up.

Electric power in the strain indicator has to be set up at least one hour before using to warm it up. The gage factor is to be dialed at 2.03 for the 125 RE - 120 rosette. When the reading for each is zeroed, the equipment turned to "test" should read ±3400 microstrains.

7) Aligning procedure

A microscope is inserted into the milling guide. Looking through the eyepiece, the operator centers the guide over the exact center of the strain gage rosette by manipulating the adjusting screws. After the adjusting screws are tightly locked, the microscope is removed and the milling bar equipped with a 0.125" drill bit (see Figure 3.11) is introduced in its place.

8) Drilling

The strain reading is set to zero for all three channels. An electric drill is adapted to the milling bar by means of a universal joint. A micrometer screw adapter (see Figures 3.11 and 3.12) is set to zero but can allow several increments of hole depth totalling 0.125". Now the screw adapter is set to 0.01" to permit a drill of 0.01" deep hole. The trigger of the drill is squeezed moderately to operate it

Figure 3.10 Adjustable swivel feet

Figure 3.11 Milling bar (with drill bit and micrometer screw adapter)

Figure 3.12 Drilling set up

Figure 3.13 Drilling under action

(see Figure 3.13) allowing the boring bar to rotate at a speed of about 200 rpm (4). When the desired depth is reached, the strain reading is made and results printed. Then the micrometer screw adapter is set to 0.02" (constant increase of depth of 0.01" each time). The operation is reiterated until reaching a hole depth of 0.12". Readings are studied to see whether they reach a maximum for each channel. Then the screw adapter is set to 0.125" (Ratio Depth/Diameter = 1.0). A depth of 0.125" is assumed to yield fully relieved strains, therefore fully relaxed residual stresses around the hole (10). Two more depth increments are made up to 0.130 and 0.140" and drilling is stopped.

8) Measuring hole diameter.

After the drilling bar is removed and the microscope inserted into the guide, the illuminator is switched on, the diameter of the finished hole is measured. The number of divisions are counted by using the micrometer and this number is multiplied by 0.00165" set by manufacturer to obtain the hole diameter. Then the ratio $r = R/R_o$ is calculated.

D) Validity of results and remarks.

The gage rosette works perfectly if it is well cemented and good solder connections are obtained. Hole drilling should be done slowly. All screws of the milling guide are to be properly tightened to prevent the drilling shaft from wobbling. Avoiding using intensive electric power in the surroundings of the strain indicator is required when strain readings are taken; otherwise an induction generated inside the strain indicator yields invalid results.

3.1.2 Drilling inside holes. Inside hole drilling is similar to outside hole drilling procedure. Figures 3.14-3.17 show the test details.

3.1.3 Experiment of tensioning a steel plate specimen for the determination of calibration coefficients. This experiment provides information to calculate calibration coefficients under a controlled condition for use in data adjustment of the results obtained from the hole-drilling technique applied to the tube.

The experiment consists of tensioning a steel plate with an initial force of known magnitude and measuring strains by means of a rosette and a reference gage, before and after drilling a hole in the plate. Then, the strains measured are analyzed to separate strains due to applied stresses from strains due to unknown residual stresses in the test plate. In this analysis, the effects of residual stresses are eliminated, and the calibration coefficients are determined by the strain-relaxation due to drilling method or by the method of separation of induced stresses and applied stresses.

A) Material and gages.

The test piece is a steel plate 5/16" thick having the shape shown in Figure 3.18.

Three gages - rosette 125RE, position A (to measure initial residual stresses) rosette 125RE, position B (to measure strain-relaxation due to drilling) and reference gage 250BG120 - is cemented to the test piece (see Figure 3.18).

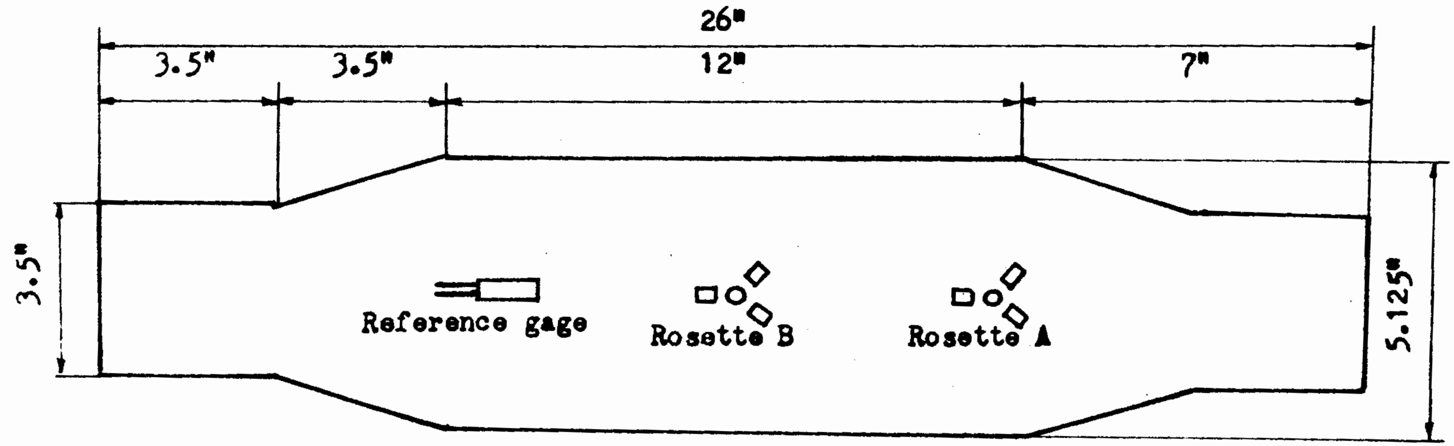
The reference gage belongs to a general purpose family of constantant strain gages widely used in experimental stress analysis.

Figure 3.14 Set up for drilling inside hole

Figure 3.15 Cementing swivel feet

Figure 3.16 Alignment set up before drilling (inside surface)

Figure 3.17 Inside hole drilling under action



<u>Figure 3.18</u> Shape of the steel plate specimen and position of gages (Rosettes A and B and reference-gage)

Its characteristics are:

Gage type: EA-06-250 BG -120

Resistance in ohms: 120.0 ± .15%

Gage factor at 75°F: 2.06 ± 0.5%

Option: L

Temperature range" -100°F to +350°F

Strain limits: 5% for gage lengths 1/8" and larger.

Cements: compatible with M-M 200 Catalyst and M-Bond 200 Adhesive or M-Bond 610.

B) Apparatus.

The apparatus used to apply a series of tensions to the test plate is a Material Testing System (MTS) with a load cell of 50 metric ton capacity, Model 661-23A-02, serial number 152. (See Figures 3.19 and 3.20.) The strains relieved from the operation of tensioning the test piece as well as of hole drilling (holes A and B) will be measured by a strain indicator (see Figures 3.4 and 3.5).

C) Experimental set up.

1) Experiment of checking the existence of residual stresses in the steel pipe.

The test plate cut to dimensions as shown in Figure 3.18 is put on a flat table. After installation of gage A on the plate (see Figure 3.21) set to zero the strain indicator.

No load is applied during this experiment.

Attach the milling guide and drill hole with increment of 0.01" hole depth each time up to total depth of 0.125" and record results (see Figure 3.22).

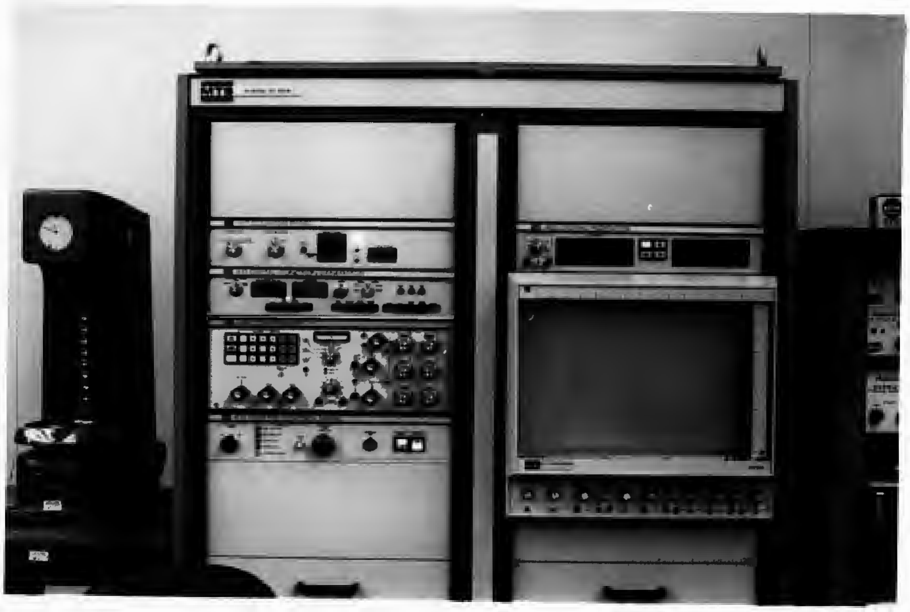


Figure 3.19 Material Testing System (MTS)

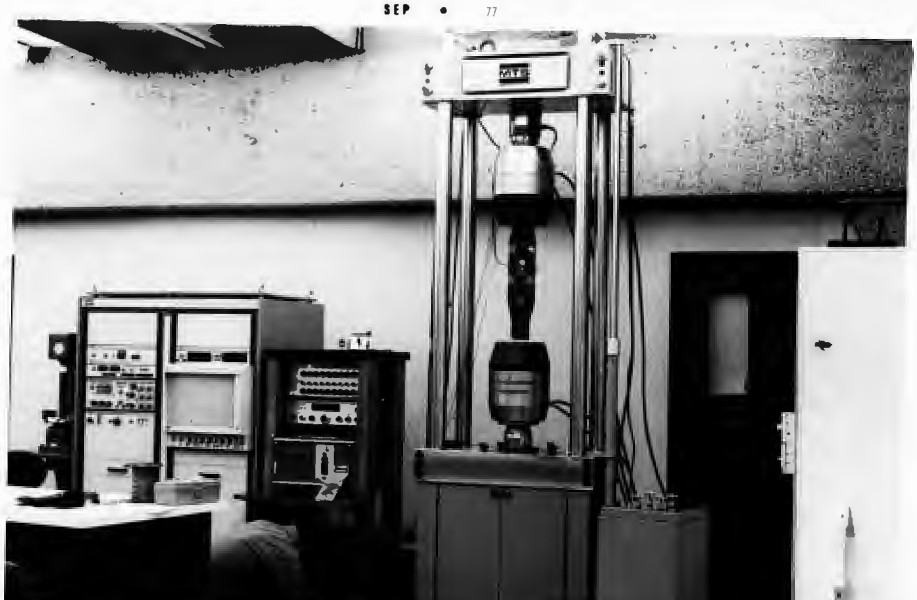


Figure 3.20 Material Testing System with load cell

Figure 3.21 Gage A for a residual stress check

Figure 3.22 Hole in gage A under drilling

The results of this experiment indicated a residual stress in the order of magnitude of 1000 psi. It will be seen later that the effect of this stress is eliminated in the determination of calibration coefficients.

2) Experiment of measuring the modulus of elasticity of the test plate steel.

Apply tensions σ_1 and σ_2 to the plate and record from the reference gage the corresponding strains ε_1 and ε_2.

The modulus of elasticity E will be given by:

$$E = (\sigma_1 - \sigma_2)/(\varepsilon_1 - \varepsilon_2)$$

The computations done in paragraph 3.2.2 show $E = 29* 10^6$ psi.

3) Experiment of determination of strain-relaxation due to drilling.

The gage B as shown in Figure 3.18, to measure disturbed strains, is cemented on the test plate and the latter is installed on the Material Testing System (see Figure 3.23).

The strain indicator is set to zero.

A tension T_1 = 26.84 kip is applied to the test piece so that the piece is under a stress σ_1 = 16.73 ksi. For each gage in the rosette, a strain reading is made, ε_a for example.

Now load T_1 is released. A hole of 0.125 in diameter and of 0.01 in depth is drilled through gage B (see Figure 3.24).

Zero strain reading, the same stress σ_1 is reapplied for the same direction 1, another strain reading is made, ε_b for example.

Figure 3.23 Setting test piece in the jaws of the Material Testing System

Compute $\Delta\varepsilon = \varepsilon_a - \varepsilon_b$. This difference between two readings before and after drilling, with the same load applied to the plate, represents the strain-relaxation due to drilling pertaining to a hole depth of 0.01 inch, for the direction 1.

The cycle of operations is reiterated until the total hole depth of 0.125 inch is reached.

The diameter of the hole is measured and checked with the expected value of 0.125 inch.

At the end of one cycle of operations, readings are made and recorded for other directions (gages) 2,3 and for the reference gage for an eventual comparison. (See Figure 3.25.)

Finally for a total hole depth of 0.125 inch is reached,

$\Delta\varepsilon_1 = \varepsilon_{a1} - \varepsilon_{b1}$ = strain relaxation due to drilling for direction 1

$\Delta\varepsilon_2 = \varepsilon_{a2} - \varepsilon_{b2}$ = strain relaxation due to drilling for direction 2

$\Delta\varepsilon_3 = \varepsilon_{a3} - \varepsilon_{b3}$ = strain-relaxation due to drilling for direction 3.

These three relaxed strains $\Delta\varepsilon_1$, $\Delta\varepsilon_2$, $\Delta\varepsilon_3$ are treated as regular measured strains to provide calibration coefficients. The testing procedure has eliminated any effect of initial residual stresses. Results will be shown in the paragraph 3.2.2 "Test results".

4) Experiment of separation of induced stresses and applied stresses.

The gage B as shown in Figure 3.18 is cemented on the plate which is mounted on the Material Testing System.

The strain indicator is set to zero. A tension T_1 =

Figure 3.24 Hole in gage B under drilling

Figure 3.25 Reference gage at left hand side

26.84 kip is applied to the plate so as to have a stress σ_1 = 16.73 ksi in it, then a strain reading is made for each direction 1,2, and 3.

While maintaining the same tension and the same stress, a hole is drilled of 0.125 in diameter and of 0.01 in depth (see Figure 3.24) and strain readings made for each direction 1, 2, and 3.

Now the tension is reduced to $T_2 = T_1/2$ = 13.42 kip so as to obtain a stress $\sigma_2 = \sigma_1/2$ = 8.37 ksi and a reading is made for each direction.

Only three trials:

Trial 1, hole depth 0.0", stress 16.73 ksi

Trial 2, hole depth 0.125", stress 16.73 ksi

Trial 3, hole depth 0.125", stress 8.36 ksi

are needed for this experiment to provide 3 corresponding strains for each direction 1, 2, and 3. These data are shown in Table 3.6.

For each direction, three strains have been recorded.

For example, for Direction 1:

Trial 1: strain 202, stress 16.73 ksi

Trial 2: strain 115, stress 16.73 ksi

Trial 3: strain 56, stress 8.36 ksi

The strain relaxed is then the difference between the strain reading before and after drilling with the plate subjected to σ_1.

Numerically: 115 - 202 = -87 microstrains.

By graphical interpolation shown in Figure 3.26, the strain caused by already present residual stresses and drilling effects is -3 microstrains.

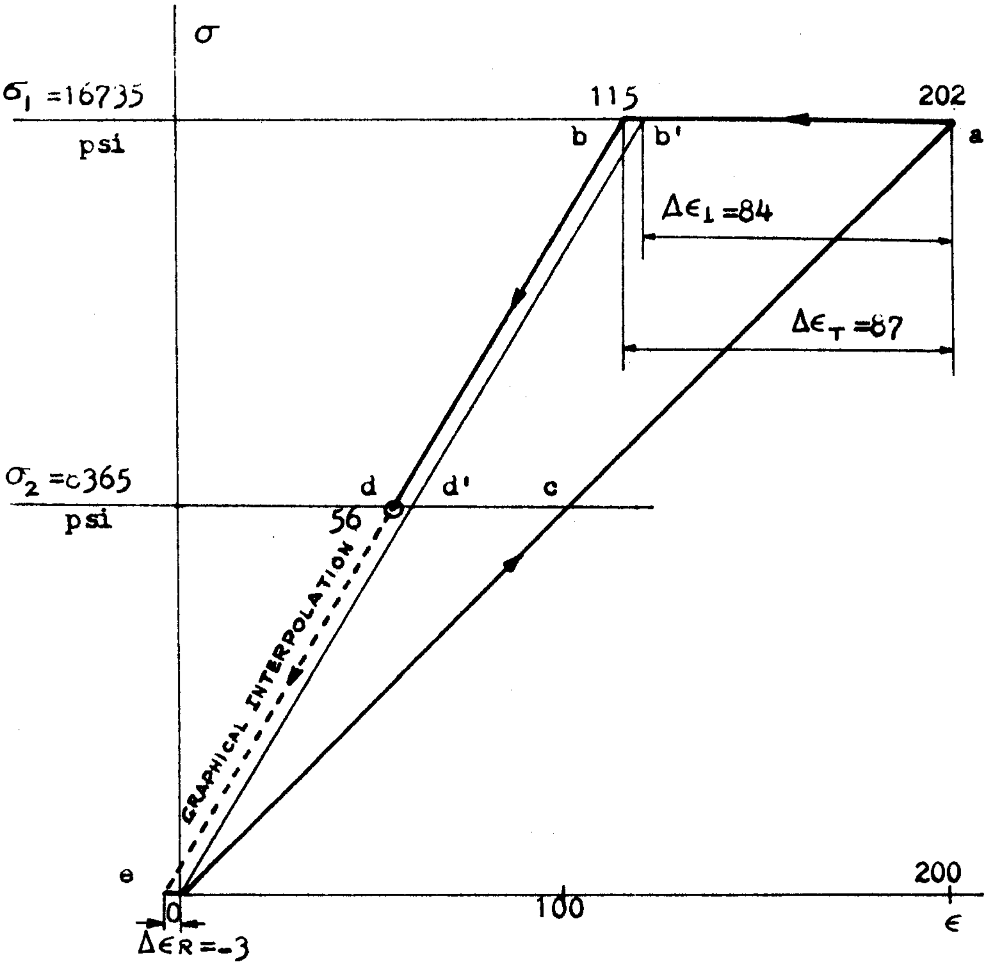


Figure 3.26. Strain separation for Direction 1 with numerical application

Strain due to applied stress $\Delta\varepsilon_1 = -87-(-3) = -84$

By the same approach, it has been found:

$$\Delta\varepsilon_2 = -143$$

$$\Delta\varepsilon_3 = -25$$

These strains due to applied stresses are treated as measured strains:

$$S = \Delta\varepsilon_1 + \Delta\varepsilon_3 = -84 - 25 = -109$$

$$D = \Delta\varepsilon_1 - \Delta\varepsilon_3 = -84-(-25) = -59$$

$$\text{Tan } 2\beta = (S - 2\Delta\varepsilon_2)/D = (-109-2(-143))/-59 = -177/59$$

$$\text{Cos } 2\beta = 0.316227$$

From equations (3a), Appendix III:

$$4A = 2S/(\sigma_1 + \sigma_2)$$

$$4B = 2D/((\sigma_1 - \sigma_2) \text{ Cos } 2\beta)$$

The experiment has provided:

$$\sigma_1 = 16730 \text{ psi}, \quad \sigma_2 = 0$$

Finally:

$$4A = 2(-109)10^{-6}/16730 = -1.30(10^{-8})$$

$$4B = 2(-59)10^{-6}/(16730 * 0.316227) = -2.23(10^{-8})$$

3.2 TEST RESULTS

3.2.1 Results from the hole drilling experiments on the steel tube.

Results from the experiments of hole drilling from the outside surface of the tube for Hole #1 are shown in detail in Tables 3.1 and 3.2. These tables give for 13 trial hole depths, measured strains for gages a,b,c, principal stresses σ_1 and σ_2 for each set of strains in three directions, longitudinal stresses σ_L and circumferential stresses

TABLE 3.1

EXPERIMENT OF OUTSIDE HOLE DRILLING

RESULTS FOR HOLE 1

Trial #	Hole Depth	Measured strains		
		Gage a	Gage b	Gage c
1	0.01"	2	10	8
2	0.02	3	14	14
3	0.03	5	22	25
4	0.04	4	24	39
5	0.05	7	31	49
6	0.06	7	30	55
7	0.07	6	30	62
8	0.08	6	30	67
9	0.09	5	27	70
10	0.10	2	26	72
11	0.11	3	30	73
12	0.125	4	27	75
13	0.13	3	26	72

TABLE 3.2

(Continued)

Trial #	Hole Depth	σ_1 psi	σ_2 psi	σ_L psi	σ_T psi	σ_L/σ_y
1	0.01"	-1190	-1799	-1744	-1214	-0.04
2	0.02	-1332	-2576	-2394	-1514	-0.06
3	0.03	-2472	-4425	-4008	-2888	-0.09
4	0.04	-3528	-6357	-5143	-4743	-0.12
5	0.05	-4730	-8134	-6677	-6197	-0.16
6	0.06	-5205	-9048	-7206	-7046	-0.18
7	0.07	-5554	-10079	-8136	-7496	-0.20
8	0.08	-5896	-10885	-8911	-7871	-0.22
9	0.09	-5880	-11353	-9461	-7781	-0.23
10	0.10	-5571	-11411	-9386	-7626	-0.23
11	0.11	-5863	-11600	-9376	-8096	-0.23
12	0.125	-6200	-11920	-10080	-8080	-0.24
13	0.13	-5711	-11530	-9541	-7701	-0.23

σ_T, ratio of σ_L to yield stress σ_y.

As the quantity of data is great, it has been simplified and shown in Table 3.3. Only data relating to depth 0.125 inch is shown, comprising strains in three directions, values of data reduction coefficients 4A and 4B, principal stresses σ_1 and σ_2, longitudinal, circumferential stresses and ratio of σ_L to yield stress σ_y.

In Table 3.4 are shown results pertaining to inside holes comprising longitudinal, circumferential stresses, ratio of σ_L to yield stress σ_y for hole depth of 0.125 inch.

In light of the data shown in these tables, an evaluation of results can be made, leading to some pertinent remarks and proving that the stress field in the welded fabricated steel tube is uniform.

A) Remarks.

A survey of values of strains for each gage in Table 3.2 shows that they gradually increase with hole depth, reach a peak for a 0.125 inch depth then slightly decrease (see Figure 3.27). This fact confirms the assertion by Bush and Kromer (2) and by S. Redner (10) that the strains relieved attain a limiting value and a maximum when hole depth approaches the hole diameter.

A second remark is that the values of stresses computed from strains measured by three gages a,b,c for one hole gradually increase and reach a maximum for ratio depth to hole radius equal to 1.0 and slightly decrease further.

Finally, values of longitudinal residual stresses found from outside and inside holes agree. In fact, for four holes number 3,4,5 and 6, the ratio of σ_L to σ_y for outside and inside holes are

TABLE 3.3

EXPERIMENT OF OUTSIDE HOLE DRILLING

DATA AND RESULTS FOR 11 HOLES

(HOLE DEPTH 0.125")

Hole #	Micro-strains			4A * ($*10^{-8}$)	4B * ($*10^{-8}$)
	ε_a	ε_b	ε_c		
1	40	27	75	-0.87	-2.50
2	-41	-25	-77	-0.88	-2.50
3	-24	-36	-41	-0.88	-2.50
4	-51	15	18	-1.05	-2.68
5	149	30	59	-0.85	-2.45
6	-31	-18	-32	-0.88	-2.50
7	-32	-90	-102	-0.85	-2.45
8	-35	-17	-21	-0.85	-2.45
9	-89	79	-104	-0.95	-2.60
A	97	101	29	-0.83	-2.37
B	20	12	92	-0.83	-2.37

* These values of 4A and 4B based on Figure 2.3

TABLE 3.3

(Continued)

Hole #	σ_1 psi	σ_2 psi	σ_L psi	σ_T psi	σ_L / σ_y
1	-6070	-12091	-10080	-8080	-0.24
2	10331	16487	16090	16130	0.24
3	6651	8122	7106	7666	0.17
4	6629	-343	792	5494	0.02
5	-31541	-17401	-30510	-18430	-0.75
6	6078	8240	6079	8239	0.13
7	12346	19184	13890	17640	0.34
8	7652	5524	5690	7486	0.14
9	6803	33828	33820	6816	0.80
A	-19484	-10878	-18390	-11970	-0.45
B	-8696	-18292	-17210	-9781	-0.42

TABLE 3.4

EXPERIMENT OF INSIDE HOLE DRILLING

RESULTS FOR FOUR HOLES

Hole #	σ_L psi	σ_T psi	σ_L / σ_y
3	6455	11160	0.16
4	- 600	- 420	-0.01
5	-29500	- 668	-0.72
6	5800	11160	0.14

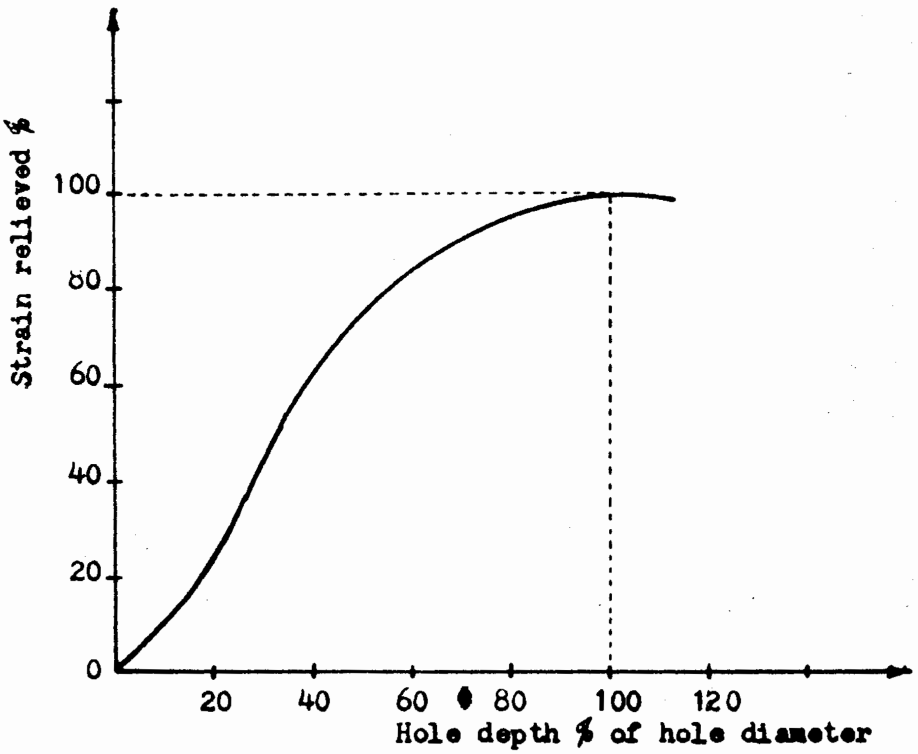


Figure 3.27 Strain relieved vs depth (Hole 1,gage c)

respectively:

0.17 and 0.16 (deviation 0.01 of σ_y)

0.02 and -0.01 (deviation 0.03 of σ_y)

-0.75 and -0.73 (deviation 0.02 of σ_y)

0.13 and 0.14 (deviation 0.01 of σ_y)

These deviations appear low and suggest a well behaved stress distribution.

B) The substantiation of the uniform field of residual stresses in the tube is now presented.

According to Kelsey (1), in a uniform field of stresses, the relationship between surface strain and hole depth is proportional to the magnitude of the stress. In other words, the incremental change in surface strain $\Delta\varepsilon_{zi}$, for an incremental change in hole depth Δz_i is proportional to the magnitude of the stress σ_{zi}.

For a material following Hooke's law

$$\varepsilon = \sigma/E$$

but for the hole drilling method, the incremental strain is not a direct measure of the average residual stress in a given increment of hole depth. Therefore, a factor of proportionality K must be introduced:

$$\Delta\varepsilon_{zi} = K\,\sigma_{zi}/E$$

Equations for the strains corresponding to the stresses in two perpendicular directions are:

$$\varepsilon_L = \sigma_L/E - \mu\sigma_T/E \qquad (1)$$

$$\varepsilon_T = \sigma_T/E - \mu\sigma_L/E \qquad (2)$$

In the hole drilling method, the above equations become:

$$\Delta\varepsilon_L = K_1\sigma_L/E - \mu K_2 \quad \sigma_T/E \tag{3}$$

$$\Delta\varepsilon_T = K_1\sigma_T/E - \mu K_2 \quad \sigma_L/E \tag{4}$$

Solving (3) and (4) in terms of $_L$ and $_T$ gives:

$$\sigma_L = E/(K_1^2 - \mu^2K_2^2) \ (K_1(\Delta\varepsilon_L) + \mu K_2 \ (\Delta\varepsilon_T)) \tag{5}$$

$$\sigma_T = E/(K_1^2 - \mu^2K_2^2) \ (K_1(\Delta\varepsilon_T) + K_2 \ (\Delta\varepsilon_L)) \tag{6}$$

Now solving equations (3) and (4) for the factors of proportionality gives:

$$K_1 = E/(\sigma_L^2 - \sigma_T^2)(\sigma_L(\Delta\varepsilon_L) - \sigma_T(\Delta\varepsilon_T)) \tag{7}$$

$$K_2 = E/\mu(\sigma_T^2 - \sigma_L^2)(\sigma_L(\Delta\varepsilon_T) - \sigma_T(\Delta\varepsilon_L)) \tag{8}$$

Using the steel plate data,

$\sigma_L = 16\ 730$ psi $\qquad \sigma_T = 0$

$\mu = 0.29 \qquad E = 29(10^6)$

$\varepsilon_L = 195(10^{-6}) \qquad \varepsilon_T = 40(10^{-6})$

and using equations (7) and (8) to compute K_1 and K_2 for the test plate:

For the hole depth from 0.0 to 0.025",

$$K_1 = \frac{29\ (10^6)}{(16730)^2 - 0}\ (16730\ (195)(10^{-6}) - 0) = 0.338$$

$$K_2 = \frac{29\ (10^6)}{0.29\ (0 - 16730^2)}\ (16730\ (40)(10^{-6}) - 0) = -0.239$$

The computed values of K_1 and K_2 for different hole depths varying from 0 to 0.125", are summarized in Table 3.5.

TABLE 3.5

VALUES OF FACTORS OF PROPORTIONALITY K_1 AND K_2 FOR THE PLATE

Hole depth	K_1	K_2
0.0 -0.025"	0.338	-0.239
0.025-0.050	0.324	-0.236
0.050-0.075	0.310	-0.281
0.075-0.100	0.296	-0.304
0.100-0.125	0.286	-0.329

Kelsey (1) has shown in his Table I for his first series of tests relating to a uniform stress field, that when his plate is subject to a tension T_1 (causing a uniform stress σ_1 in his plate) and subject later to a tension T_2 (causing a uniform stress σ_2) the respective longitudinal and transverse strains ε_{L1}, ε_{T1} and ε_{L2} ε_{T2} are bound by the relationships:

$$\frac{\varepsilon_{L1}}{\varepsilon_{L2}} = \frac{\varepsilon_{T1}}{\varepsilon_{T2}} = \frac{\sigma_1}{\sigma_2} \quad (= 2.0 \text{ for his case})$$

A first sample of calculation to prove the equality of the above ratios is taken from a study by Kelsey (1, Table I):

For the depth of 0.250" of a hole drilled in a uniform stress field:

ε_{L1} = -376 microstrains, ε_{T1} = 122 microstrains, σ_{L1} = 9700 psi

ε_{L2} = -800 - , ε_{T2} = 249 - , σ_{L2} = 19400 psi

Ratios:

$$\frac{\varepsilon_{L2}}{\varepsilon_{L1}} = 2.1, \qquad \frac{\varepsilon_{T2}}{\varepsilon_{T1}} = 2.0, \qquad \frac{\sigma_{L2}}{\sigma_{L1}} = 2.0$$

A second sample of calculation to substantiate the equality of these ratios is taken from the case of our steel plate subject to uniform tensions T_1 and T_2, data read from the Hole B with a hole depth of 0.125":

ε_{L1} = 82 microstrains, ε_{T1} = 27 microstrains, σ_{L1} = 8370 psi

ε_{L2} = 165 - , ε_{T2} = 55 - σ_{L2} = 16730 psi

Ratios:

$$\frac{\varepsilon_{L2}}{\varepsilon_{L1}} = 2.0, \qquad \frac{\varepsilon_{T2}}{\varepsilon_{T1}} = 2.0, \qquad \frac{\sigma_{L2}}{\sigma_{L1}} = 2.0$$

The values of these ratios of strains and stresses show good agreement.

To prove that a non-uniform stress field does not have these characteristics: that is =

$$\frac{\varepsilon_{L2}}{\varepsilon_{L1}} \neq \frac{\varepsilon_{T2}}{\varepsilon_{T1}} \neq \frac{\sigma_{L2}}{\sigma_{L1}}$$

reference is again made to Kelsey, Table II (1) for a non-uniform stress field studied by Kelsey shows that the ratios $\frac{\varepsilon_{L2}}{\varepsilon_{L1}}$, $\frac{\varepsilon_{T2}}{\varepsilon_{T1}}$, $\frac{\sigma_{L2}}{\sigma_{L1}}$ are not equal (see Table 3.6).

TABLE 3.6

DATA FROM A NON-UNIFORM STRESS FIELD

Hole depth	ε_L	$\frac{\varepsilon_{L(i+1)}}{\varepsilon_{Li}}$	ε_T	$\frac{\varepsilon_{T(i+1)}}{\varepsilon_{Ti}}$	σ_L (a)	$\frac{\sigma_{L(i+1)}}{\sigma_{Li}}$	σ_L (b)	$\frac{\sigma_{L(i+1)}}{\sigma_{Li}}$
0.025"	-122		20		18600		18700	
		2.24		2.0		0.89		0.88
0.050	-274		40		16600		16500	
		1.50		1.47		0.88		0.92
0.075	-411		59		14600		15200	
		1.21		1.27		0.86		0.92
0.100	-498		75		12600		14000	
		1.10		1.21		0.84		0.87
0.125	-548		91		10600		12200	
		1.06		1.13		0.81		1.21
0.150	-583		103		8600		14800	
		1.04		1.09		0.76		1.21
0.175	-607		112		6600		17900	

(a) Values computed directly from measured strains

(b) Values calculated using K_1 and K_2 from a uniform stress field.

In order to substantiate that the field of residual stresses present in the steel tube is uniform, through the depth, we first calculate the stresses σ_{Li} and $\sigma_{L(i+1)}$ corresponding to hole depths

Z_i and Z_{i+1} in the steel tube, using K_1 and K_2 obtained from the test plate and the corresponding hole depths in the test plate.

The second step is to compute the ratios $\frac{\varepsilon_{L(i+1)}}{\varepsilon_{Li}}$, $\frac{\varepsilon_{T(i+1)}}{\varepsilon_{Ti}}$ and $\frac{\sigma_{L(i+1)}}{\sigma_{Li}}$ and compare.

Sample calculation:

<u>Step 1</u>. <u>Trial 1.</u> For hole depth of 0.25" in the test plate, K_1 = 0.338, K_2 = -0.239 (see Table A). With these values of K_1 and K_2 (from the plate) and values of $\varepsilon_L = -3(10^{-6})$, $\varepsilon_T = 10^{-6}$ of the same hole depth of the Hole 4 in the tube, compute σ_{L1}, for this Hole:

$$\sigma_{L1} = \frac{29(10^6)}{0.338^2 - (0.29*0.239)^2}[0.338(-3) + (0.29)(-0.239)(1)](10^{-6})$$

$$= -287 \text{ psi}$$

<u>Trial 2</u>. For the hole depth of 0.050" in the test plate, with the values of K_1 = 0.324, K_2 = -0.263 and the values of ε_L = $-15(10^{-6})$, $\varepsilon_T = 4(10^{-6})$ of the same hole depth of the Hole 4 in the tube, compute σ_{L2} for this Hole:

$$\sigma_{L2} = \frac{29(10^6)}{0.324^2 - (0.29*-0.263)^2}[(0.234)(-15) + (0.29)(-0.263)(4)](10^{-6})$$

$$= -1510 \text{ psi.}$$

<u>Step 2</u>. Compute the ratios:

$$\frac{\sigma_{L2}}{\sigma_{L1}} = \frac{-1510}{-287} = 5.2, \qquad \frac{\sigma_{T2}}{\sigma_{T1}} = \frac{4}{0.8} = 5.0$$

$$\frac{\varepsilon_{L2}}{\varepsilon_{L1}} = \frac{-15}{-3} = 5.0$$

The deviation: $(5.2-5.0)/5.0 = 0.04$ of $\varepsilon_{L2}/\varepsilon_{L1}$ appears acceptable.

The computations of σ_L and σ_T and the required ratios are summarized in Table 3.7.

The ratios of $\frac{\varepsilon_{L(i+1)}}{\varepsilon_{Li}}$, $\frac{\varepsilon_{T(i+1)}}{\varepsilon_{Ti}}$, $\frac{\sigma_{L(i+1)}}{\sigma_{Li}}$ show close agreement, and the uniform stress field assumed in Chapter II is substantiated by numerical calculation.

3.2.2 <u>Tensioning and calibration experiment</u>.

A) Results of the test.

Data derived from the experiment of strain-relaxation due to drilling are condensed in Table 3.8. These data will be used in section 3.2.2.c to compute the values of calibration coefficients 4A and 4B.

Data pertaining to the experiment of separation of induced stresses and applied stresses are shown in Table 3.9. Calibration coefficients 4A and 4B are computed from these data and described in section 3.2.2.d.

B) Computation of the value of modulus of elasticity.

The modulus of elasticity in a strain-stress curve is given by the slope of the straight part of the curve. As the maximum stress applied to the plate is 0.40 F_y (within the limit of elasticity) Hooke's law is valid and:

$$E = \sigma/\varepsilon \qquad \text{or} \qquad E = (\sigma_2 - \sigma_1)/(\varepsilon_2 - \varepsilon_1)$$

A measurement of the thickness and width of the plate provides:

TABLE 3.7

RESULTS OF APPLICATION TO TUBE DATA. (HOLE 4)

Trial	Hole depth	ε_L (μ strain)	ε_T (μ strain)	σ_L psi	σ_T psi	$\frac{\varepsilon_{L(i+1)}}{\varepsilon_{L(i)}}$	$\frac{\sigma_{L(i+1)}}{\sigma_{L(i)}}$	$\frac{\varepsilon_{T(i+1)}}{\varepsilon_{T(i)}}$	$\frac{\sigma_{T(i+1)}}{\sigma_{T(i)}}$
1	.025"	-3	0.8	-287	145				
						5.0	5.2	5.0	4.9
2	.050	- 5	4	-1510	714				
						1.6	1.7	1.5	1.7
3	.075	-24	6	-2570	1237				
						1.1	1.1	1.2	1.2
4	.100	-26	7	-3019	1585				
						1.2	1.2	1.3	1.3
5	.125	-30	9	-3765	2151				

TABLE 3.8

EXPERIMENT OF TENSIONING & CALIBRATION

DATA FROM STRAIN-RELAXATION METHOD

Trial #	Hole Depth	Hole Diameter	Measured micro-strains			Reference gauge	Stress Applied (ksi)
			ε_1	ε_2	ε_3		
1	0.0	0.0	202	533	193	572	16.73
2	.025	.20	197	515	185	572	16.73
3	.050	.40	182	474	171	572	16.73
4	.075	.60	171	442	161	572	16.73
5	.100	.80	165	423	156	570	16.73
6	.125	1.00	162	407	155	571	16.73

TABLE 3.9

EXPERIMENT OF TENSIONING AND CALIBRATION

DATA FROM METHOD OF STRAIN RELAXATION

Trial #	Hole Depth	Hole Depth Diameter	Measured mirco-strains ε_1	ε_2	ε_3	Reference gauge	Stress applied (ksi)
0	0	0	0	0	0	0	0
1	0.0"	0.0	202	533	193	572	16.73
6	0.125	1.0	115	395	172	570	16.73
7	0.125	1.0	56	200	88	285	8.36

Thickness: 5/16" or 0.3125"

Width: 5 1/8" or 5.125"

True cross-sectional area: 1.60 square inches

As applied tensions were 26.8 kip and 13.4 kip and parallel to longitudinal axis, relative stresses would be respectively:

$$\sigma_{L2} = 26.8/1.6 = 16.73 \text{ ksi}$$

$$\sigma_{L1} = 13.4/1.6 = 8.37 \text{ ksi}$$

The corresponding strains measured were:

$$\varepsilon_{L2} = 574 \text{ microstrains}$$

$$\varepsilon_{L1} = 286 \text{ microstrains}$$

Finally:

$$E = (16.73 - 8.37)10^3/(574-286)10^{-6}$$

$$= 29 \times 10^6 \text{ psi}$$

C) Results from method of strain-relaxation due to drilling.

From Table 3.5, the strain-relaxation due to drilling (2) equals the difference of strain readings before and after drilling (depth 0.125") when the test plate is under the same applied load (16.73 ksi):

For direction 1:

$= -202 + 162 = -40$ microstrains

For direction 2:

$= -533 + 407 = -126$ microstrains

For direction 3:

$= -193 + 155 = -38$ microstrains

These strains of strain-relaxation due to drilling are treated as measured strains (see equations (3a), Appendix III):

$$S = \Delta\varepsilon_1 + \Delta\varepsilon_3 = -78$$

$$D = \Delta\varepsilon_1 - \Delta\varepsilon_e = -2$$

$$\text{Tan } 2\beta = (S-2\Delta\varepsilon_2)/D = -87$$

$$\text{Cos } 2\beta = 0.011493$$

From equations (3a) (Appendix III)

$$4A = 2S/(\sigma_1 + \sigma_2)$$

$$4B = 2S/((\sigma_1 - \sigma_2) \text{ Cos } 2\beta)$$

The experiment thus provides: $\sigma_1 = \sigma_L = \dfrac{\text{Tension}}{\text{Area}}$,

$$\sigma_1 = 26.9/(0.3125*5.125) = 16.73 \text{ ksi or } 16730 \text{ psi}$$

$$\sigma_2 = \sigma_T = 0.$$

Therefore, calibration coefficients are:

$$4A = 2(-78)10^{-6}/16730 = -0.93*10^{-8}$$

$$4B = 2(-2)10^{-6}/((16730)(0.011493)) = -2.08*10^{-8}$$

D) Results from method of separation of induced stresses and applied stresses.

Using the method of separating induced stresses and applied stresses, it is found by graphical procedure:

For direction 1: (see Figure 3.26)

Total strain relaxes: -(202-115) = -87 microstrains

Strain due to residual stresses: -3

Strain due to applied stress only: $\Delta\varepsilon_1 = -87-(-3) = -84$

For direction 2:

Total strain relaxed: -(533 - 395) = -138

Strain due to residual stresses: +5

Strain due to applied stress: $\Delta\varepsilon_2 = -138-(5) = -143$

For direction 3:

Total strain relaxed: $-(193-172) = -1$

Strain due to residual stresses: $+4$

Strain due to applied stress: $\Delta\varepsilon_3 = -21-(4) = -25$

These strains due to applied stresses are treated as measured strains:

$$S = \Delta\varepsilon_1 + \Delta\varepsilon_3 = -109$$

$$D = \Delta\varepsilon_1 - \Delta\varepsilon_3 = -59$$

$$\text{Tan } 2\beta = (S - 2\Delta\varepsilon_2)/D = -177/59$$

$$\text{Cos } 2\beta = 0.316227$$

From equations (3a)

$$4A = 2S/(\sigma_1 + \sigma_2)$$

$$4B = 2D/((\sigma_1 - \sigma_2) \text{ Cos } 2\beta)$$

It has been found by experiment that:

$$\sigma_1 = 16730 \text{ psi}, \qquad \sigma_2 = 0$$

Therefore:

$$4A = 2(-109)10^{-6}/16730 = -1.30*10^{-8}$$

$$4B = 2(59)10^{-6}/(16730*0.316227) = -2.23*10^{-8}$$

E) Summary of results.

In this investigation, four ways of computing the values of 4A and 4B were used.

1) For each hole drilled, its radius R_o is measured and the ratio r of the distance of the gage to center R(R=0.202" for Rosette 125RE) to radius of hole drilled R_o ($r=R/R_o$) is evaluated. In Figure 2.3 the line of abscissa r parallel to vertical axis, cuts the curve 4A at point a and the curve 4B at point b, the

ordinates of a and b are the values of 4A and 4B. The values of 4A and 4B of the 11 drilled holes are summarized in Table 3.10.

It is of interest to note that in Table 3.10 4A and 4B differ for different holes, because they are determined from equations which are functions of r, $r=R/R_o$, r varies with R_o where R_o is the measured radius of the hole drilled.

TABLE 3.10

VALUES OF 4A AND 4B COMPUTED BY CURVES FROM FIGURE 2.3

Hole	4A (10^{-8})	4B (10^{-8})
1	-0.87	-2.50
2	-0.87	-2.50
3	-0.85	-2.45
4	-1.05	-2.48
5	-0.85	-2.45
6	-0.88	-2.50
7	-0.85	-2.45
8	-0.85	-2.45
9	-0.95	-2.60
A	-0.83	-2.37
B	-0.83	-2.37

2) From equation (1)

$$\varepsilon_r = -\sigma_1 \frac{1+\mu}{2E} [1/r^2 - 3/r^4 \text{ (Cos } 2\alpha) + 4/(1+\mu) \text{ (Cos } 2\alpha)$$

and

$$\varepsilon_r = (A + B \text{ Cos } 2\alpha)\sigma_1$$

By identifying the similar terms, it is found:

$$A = -(1+\mu)/(2E*r^2)$$

where $\mu = 0.29$, $E = 29(10^6)$,

$$r = 0.202*2/0.125 = 3.23$$

Then,

$$4A = -\frac{2(1+0.29)}{(3.23)^2(29)(10^6)} = -0.86(10^{-8})$$

and

$$B = (1+\mu)/2E[3/r^4-4/((1+\mu)(r^2))]$$

Thus,

$$4B = \frac{2(1+0.29)}{29(10^6)}\ \frac{3}{(3.23)^4} - \frac{4}{1.29(3.23)^2}$$

$$4B = -2.41(10^{-8})$$

3) The values of 4A and 4B found experimentally by the method of strain-relaxation

$$4A = -1.30(10^{-8})$$

$$4B = -2.08(10^{-8})$$

according to the computations shown in paragraph 3.2.2.c (strain-relaxation).

4) The values of 4A and 4B found experimentally by the method of strain separation

$$4A = -1.30(10^{-8})$$

$$4B = -2.23(10^{-8})$$

according to the computations shown in paragraph 3.2.2.d (strain-separation).

For a better comparison between various calibration coefficients obtained from measurements or computations, results are summarized in Table 3.11.

TABLE 3.11

SUMMARY OF 4A AND 4B VALUES

Case #	Measurement or Method Used	4A (10^{-8})	4B (10^{-8})
1	Measurement of hole diameter and reading from curves in Figure 2.3	-0.83 to -1.05	-2.37 to -2.68
2	Computation from equation (1) with theoretical hole diameter 0.125"	-0.86	-2.41
3	Computation using method of strain-relaxation due to drilling	-0.93	-2.08
4	Computation using method of separation of induced stresses and applied stresses	-1.30	-2.23

In the light of the data in the table, it is revealed that most values of 4A and 4B computed are reasonable and within the range of acceptable measured values. The criterion used for appreciating the degree of adequacy of each pair of 4A and 4B is likely the quality of results from the balance of forces and moments of residual stresses. As the system of longitudinal residual stresses present in a circular section of the tube must have a state of force equilibrium and of bending moment equilibrium (3) at the same time, there currently is a deviation in the results given by the computation of balancing forces and moments of residual stresses; this deviation derives from accumulation of errors during experimentation. Because force and moment balancing will be described in Chapter IV, it is appropriate to evaluate the results of calibration coefficients after obtaining results of balancing.

F) Application of calibration coefficients to tube data.

After obtaining all pairs of calibration coefficients 4A and 4B for each case as shown in Table 3.7, it is necessary to apply these coefficients to tube data and determine the corresponding longitudinal residual stresses, circumferential residual stresses and ratios of longitudinal residual stresses to yield stress, and to plot curves of the distribution. Curves are built with d/R on the abscissa (d = distance of hole to the weld, R = radius of the tube) and σ_L/σ_y on the ordinate.

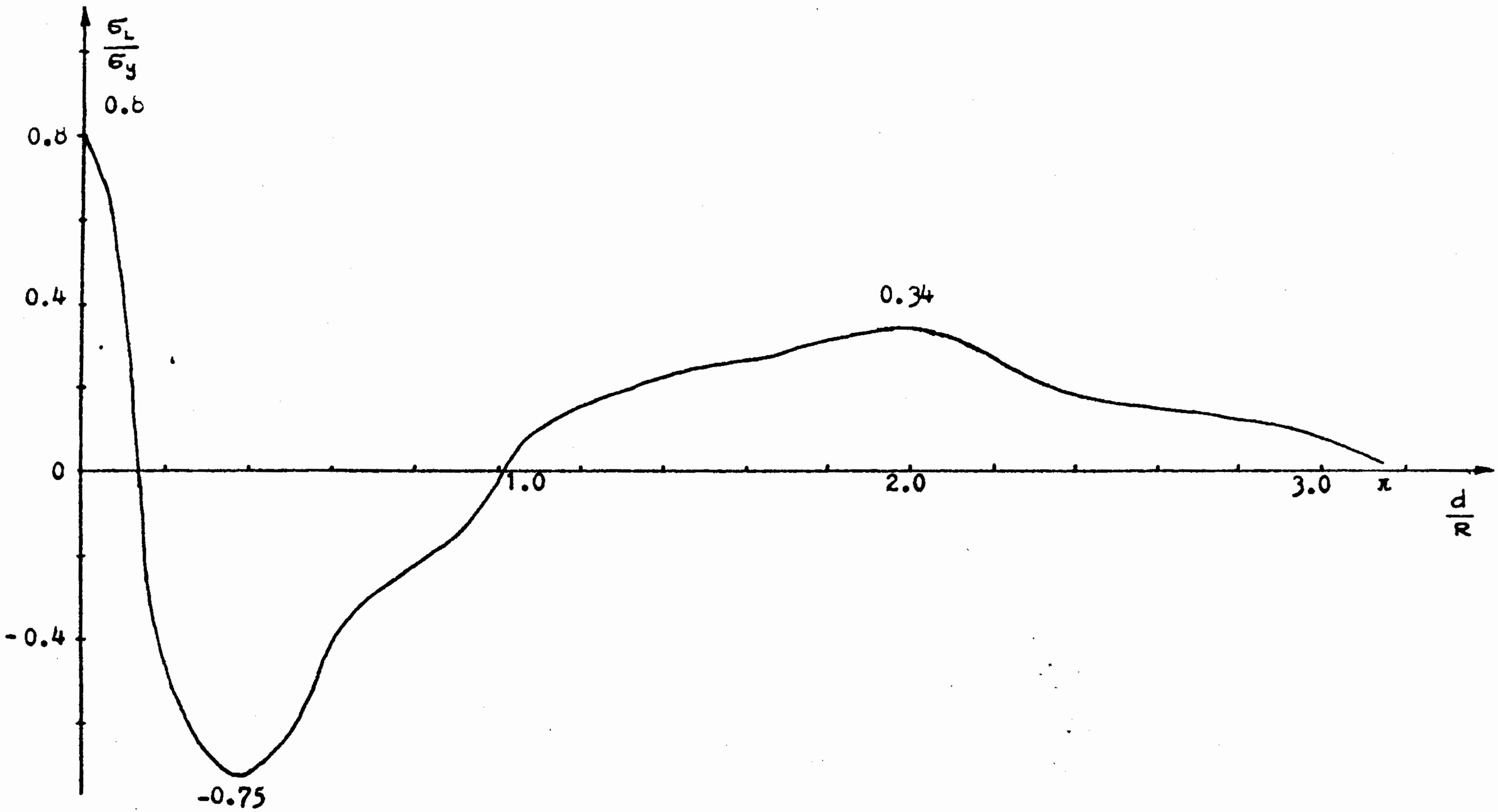
Case 1. Results relating to initial values of longitudinal and circumferential stresses derived from measured strains, are given in Table 3.3. The relative calibration coefficients 4A and 4B are obtained from the curves in Figure 2.3.

The corresponding curve of σ_L/σ_y is drawn in Figure 3.28.

Case 2. Results pertaining to data reduction coefficients 4A and 4B computed from equation (1), are shown in Table 3.12.

The corresponding curve is drawn in Figure 3.29.

Case 3. Results pertaining to calibration coefficients obtained from method of strain-relaxation due to drilling, are condensed in Table 3.13.



<u>Figure 3.28</u> Curve of initial values of measured longitudinal residual stresses

TABLE 3.12

RESULTS WITH COMPUTED CALIBRATION COEFFICIENTS

$$4A = -0.86(10^{-8})$$

$$4B = -2.41(10^{-8})$$

Hole #	σ_L psi	σ_T psi	σ_L / σ_y
1	-10220	- 8149	-0.25
2	10900	16540	0.27
3	7268	7849	0.18
4	1223	6451	0.03
5	-30330	-18040	-0.75
6	6205	8446	0.15
7	13670	17490	0.33
8	5599	7424	0.14
9	37010	7877	0.90
A	-18390	-11970	-0.45
B	-16670	- 9372	-0.41

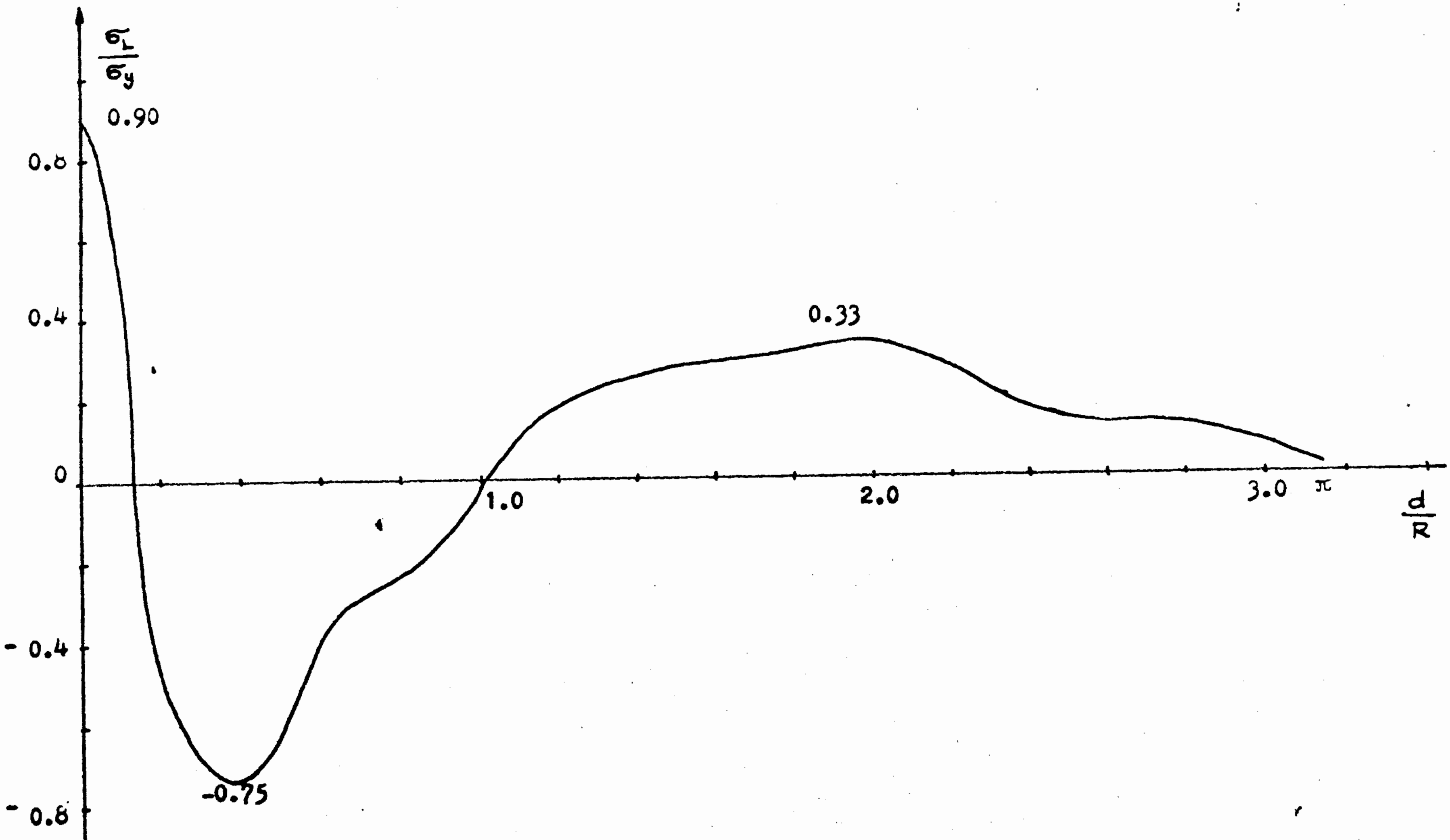


Figure 3.29 Curve of longitudinal residual stresses recalculated with data reduction coefficients: $4A = -0.86(10^{-8})$, $4B = -2.41(10^{-8})$

TABLE 3.13

RESULTS FROM METHOD OF STRAIN-RELAXATION
DUE TO DRILLING

CALIBRATION COEFFICIENTS

$$4A = -0.93(10^{-8})$$

$$4B = -2.08(10^{-8})$$

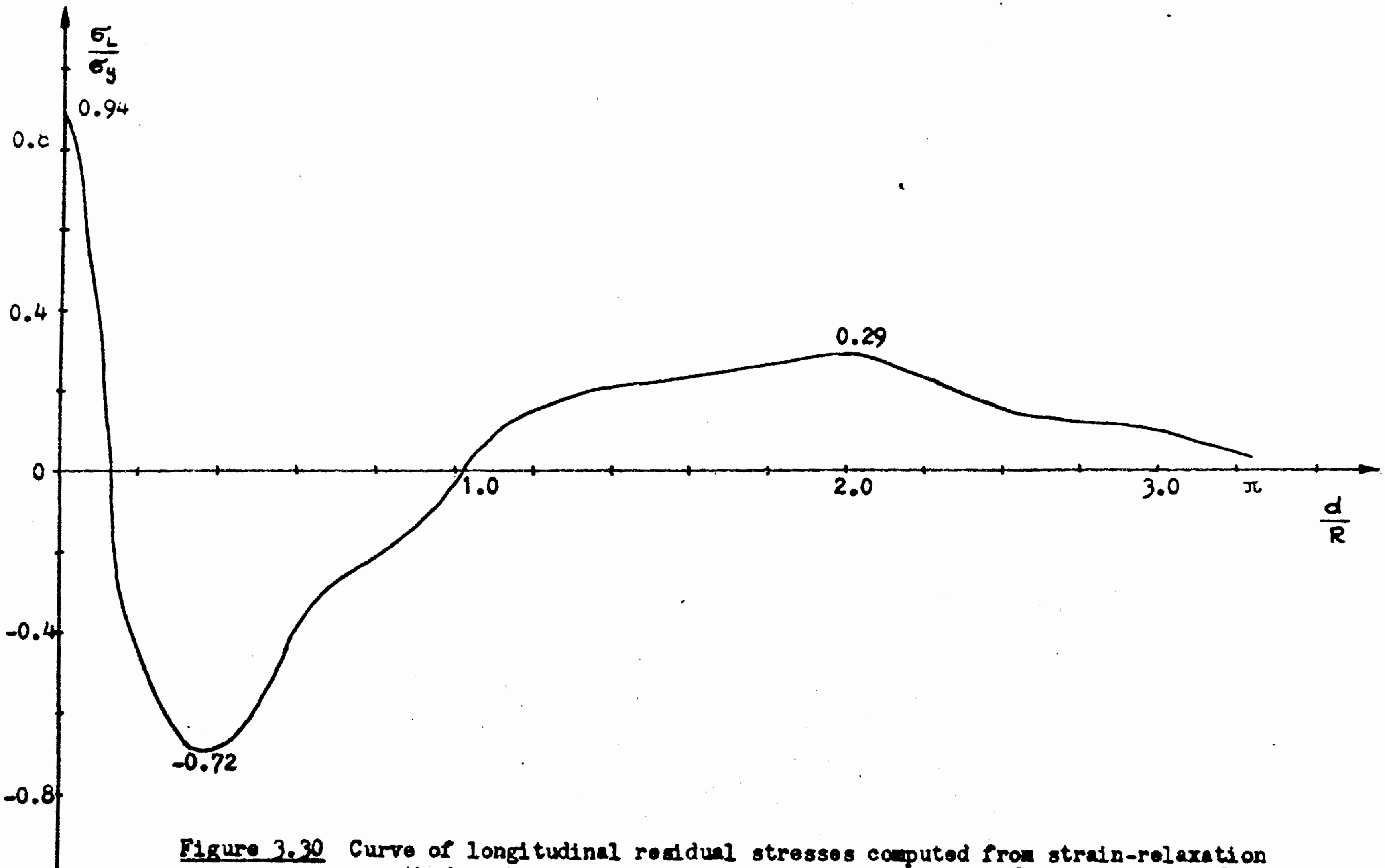
Hole #	σ_L psi	σ_T psi	σ_L / σ_y
1	- 9697	- 7293	-0.24
2	9400	15900	0.23
3	7326	6653	0.16
4	519	6577	0.01
5	-29480	-15250	-0.72
6	5476	8072	0.13
7	12200	16620	0.29
8	4964	7080	0.12
9	37630	3878	0.94
A	-17200	- 9895	-0.42
B	-16270	- 7812	-0.40

The curve is shown in Figure 3.30.

Case 4. Results relating to calibration coefficients calculated from method of separation of induced stresses and applied stresses, are shown in Table 3.14.

The curve is given in Figure 3.31.

A preliminary survey of these curves shows that the one in case 4 is likely to lead to good results in balancing forces and moments because of the reduced tension and a slight compressive area at hole number 4 which helps in moment balancing.



<u>Figure 3.30</u> Curve of longitudinal residual stresses computed from strain-relaxation method with calibration coefficients: $4A=-0.93(10^{-8})$, $4B=-2.08(10^{-8})$

TABLE 3.14

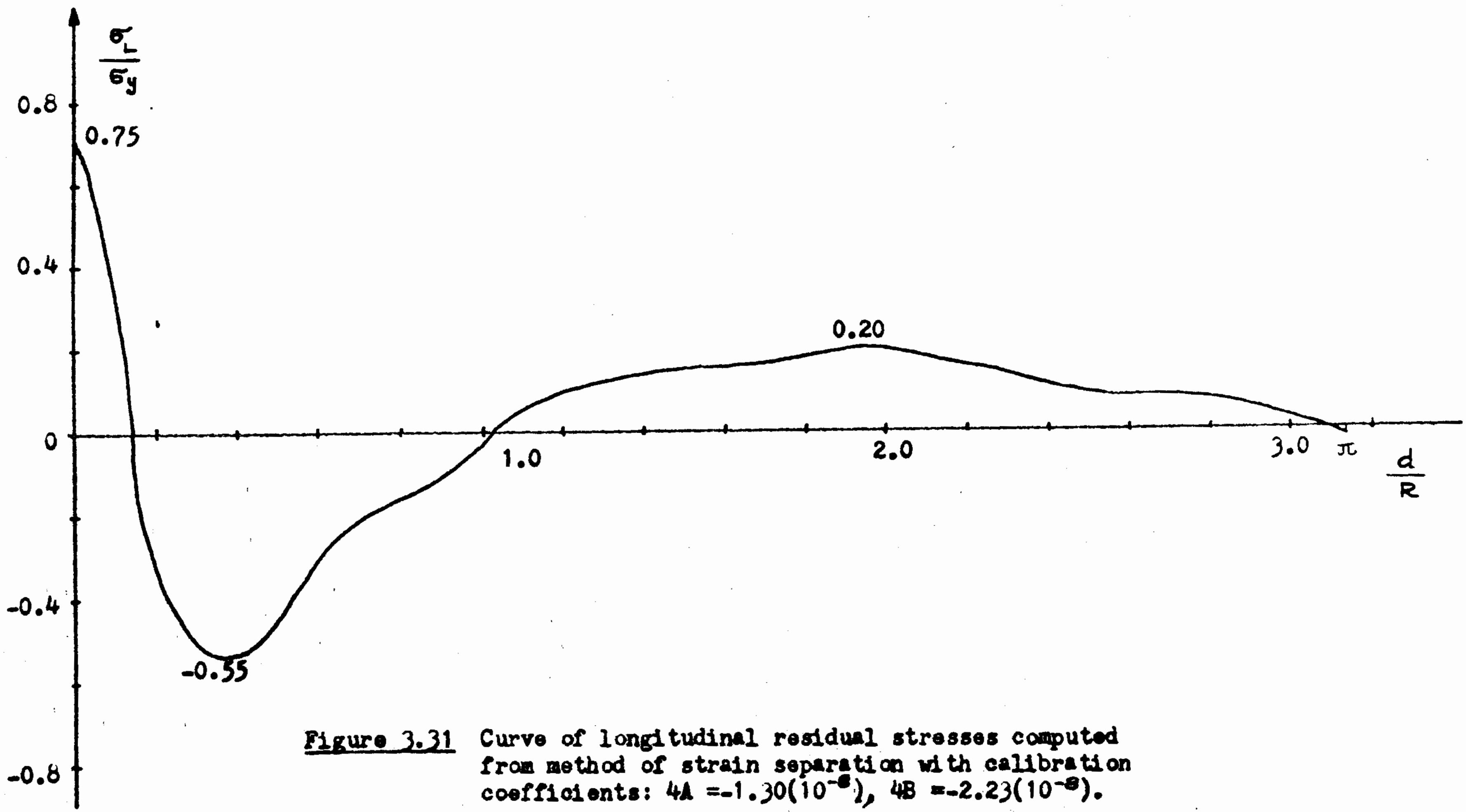
RESULTS FROM METHOD OF STRAIN SEPARATION

CALIBRATION COEFFICIENTS

$$4A = -1.30(10^{-8})$$

$$4B = -2.23(10^{-8})$$

Hole #	σ_L psi	σ_T psi	σ_L/σ_y
1	- 7198	- 4956	-0.17
2	6028	12130	0.15
3	4686	5314	0.11
4	- 287	5364	-0.01
5	-22640	- 9363	-0.55
6	3635	6057	0.09
7	8245	12370	0.20
8	3321	5294	0.08
9	30590	- 8938	0.75
A	-13100	- 6284	-0.32
B	-12560	- 4669	-0.31



<u>Figure 3.31</u> Curve of longitudinal residual stresses computed from method of strain separation with calibration coefficients: $4A = -1.30(10^{-8})$, $4B = -2.23(10^{-8})$.

CHAPTER IV

BALANCE OF FORCES AND MOMENTS

As a means of checking the correctness of results of the measurement of residual stresses by the hole-drilling technique a balance of forces and moments pertaining to longitudinal residual stresses is in equilibrium in the steel tube, the summation of forces and moments with respect to an axis must equal zero (3). It is convenient to show the balancing of forces and moments, using calibration coefficients obtained from theory and published literature and experimentally determined.

4.1 BALANCING FORCES AND MOMENTS OF RESULTS USING ANALYTICALLY DERIVED CALIBRATION COEFFICIENTS

The development of an analytical approach is presented first, this is then followed by a flow chart and a computer program used to perform the calculation.

4.1.1 Analytical approach.

A) Pattern of longitudinal residual stress distribution.

A cross-sectional area, two feet from edge of the steel tube, perpendicular to longitudinal axis shown in Figure 4.1 shows the pattern of longitudinal residual stress distribution. These stresses have been obtained from measured strains, with variable 4A and 4B, case 1, in Table 3.11. Positive ordinates outside the circle denote tensile stresses and negative ordinates inside the circle represent compressive stresses. Therefore positive areas denote tension and

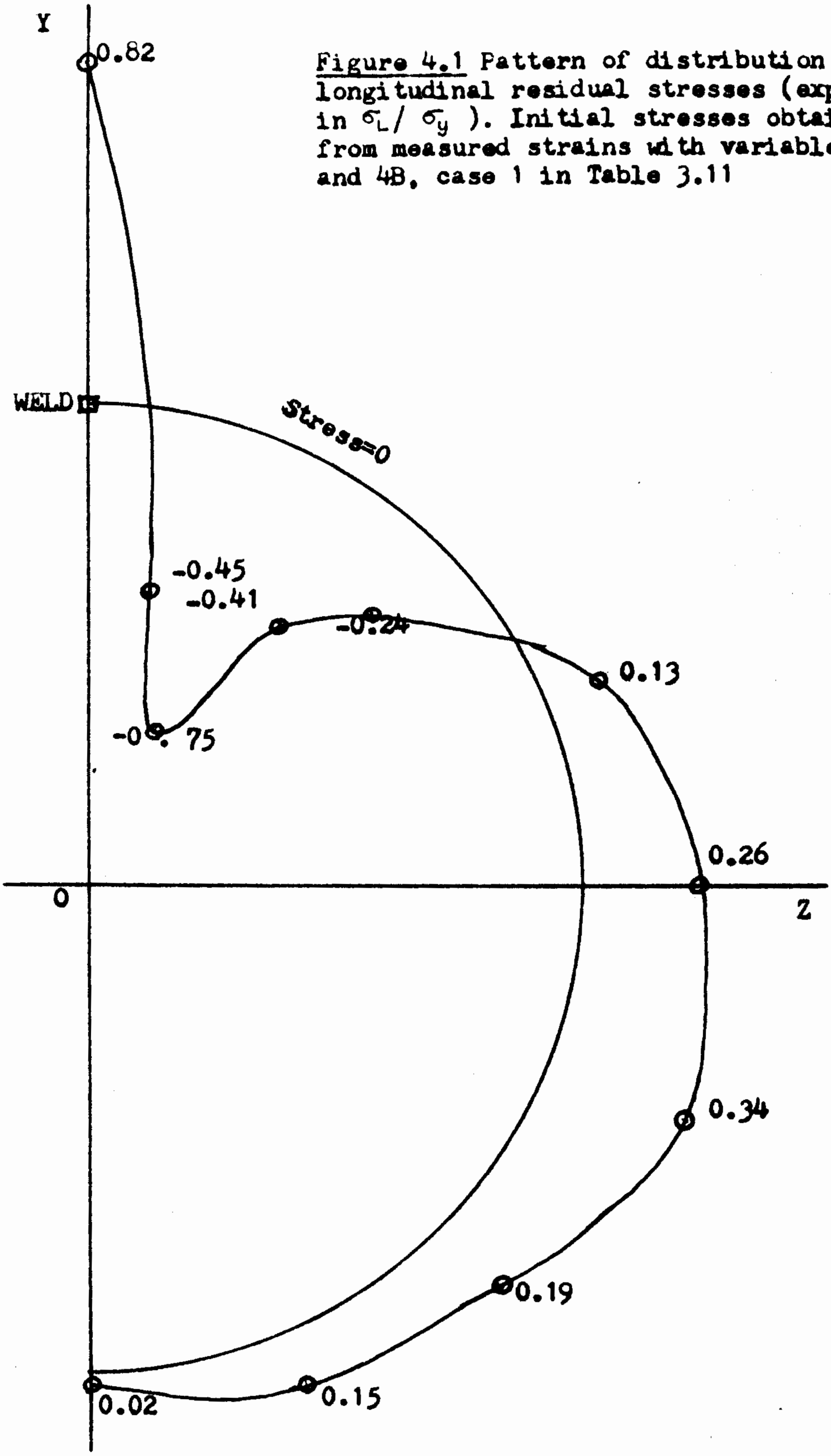


Figure 4.1 Pattern of distribution of longitudinal residual stresses (expressed in σ_L / σ_y). Initial stresses obtained from measured strains with variable 4A and 4B, case 1 in Table 3.11

negative areas represent compression.

An attempt will be made to calculate the summation of forces and summation of moments about Z axis shown in Figure 4.1.

As to the sign of moments, it is useful to note that:

-For the part above Z axis:

a moment of a positive area is positive,

a moment of a negative area is negative.

-For the part below the Z axis:

a moment of a positive area is negative,

a moment of a negative area is positive,

using the convention of positive sign for a clockwise moment.

As to the summation of forces, the computer program defines it by adding algebraically the positive areas and negative areas.

As to the summation of moments of forces in the tube, an algebraic addition is performed on partial moments of area, the sign of each moment relating to the area above or below the Z axis as defined previously.

It is understood that the moment about Z axis is the product of the volume of forces (area of forces times the thickness of the tube) and the lever arm which is the distance from the center of gravity of the area to the Z axis.

B) Area and center of gravity.

The procedure programmed is a form of variable step numerical integration in which the user, in preparing data for the computer program, defines and calculates the area and the center of gravity of the force segments. The Figure 4.2 shows a possible choice of

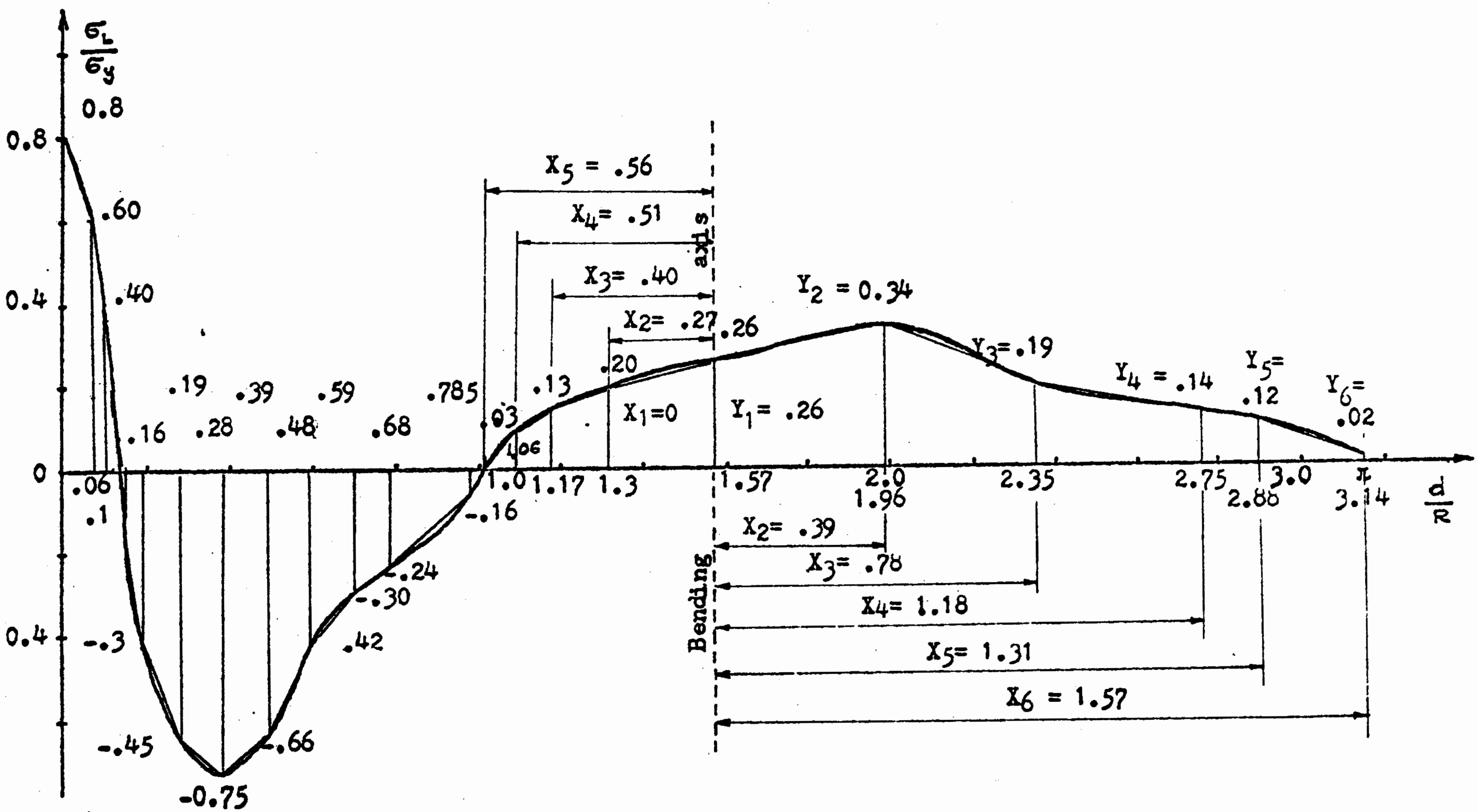


Figure 4.2 Segmental division for preparing data for the computer program

segmental division.

The following cases give the details of the signs and quantities required as input.

Case 1. (see Figure 4.3)

Distance of center of gravity of the rectangle to Y axis:

$$CG1 = (X_1 + X_2)/2$$

Distance of center of gravity of the triangle to Y axis:

$$CG2 = (2X_2 + X_1)/3$$

$$\text{Area A1} = (X_2 - X_1)/Y_1$$

$$\text{Area A2} = (X_2 - X_1)(Y_2 - Y_1)/2$$

Case 2. (see Figure 4.4)

$$CG1 = (X_1 + X_2)/2$$

$$CG2 = (2X_1 + X_2)/3$$

$$A1 = (X_2 - X_1)Y_1$$

$$A2 = (X_2 - X_1)(Y_2 - Y_1)/2$$

Case 3. (see Figure 4.5)

$$X_o = (X_2 - X_1)Y_1/(Y_2 + Y_1)$$

$$A1 = X_oY_1/2$$

$$CG1 = X_1 + X_o/3$$

$$A2 = (X_2 - X_1 - X_o)Y_2/2$$

$$CG2 = (2X_2 + X_1 + X_o)/3$$

Case 4. (see Figure 4.6)

$$X_o = -(X_2 - X_1)Y_1/(Y_2 - Y_1)$$

$$CG1 = X_1 + X_o/3$$

$$A1 = X_oY_1/2$$

$$CG2 = (2X_2 + X_1 + X_o)/3$$

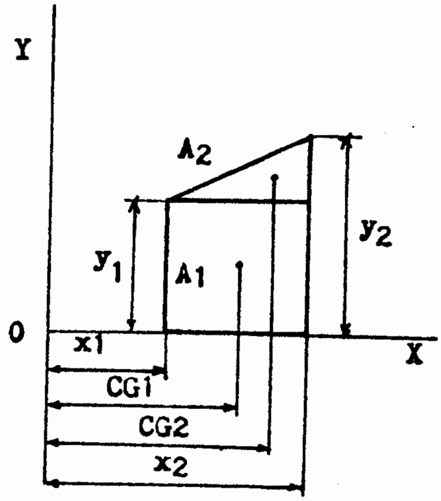


Figure 4.3 Area and center of gravity.
Case 1

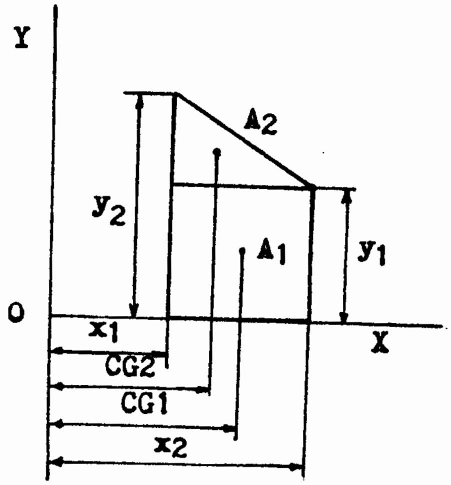


Figure 4.4 Case 2

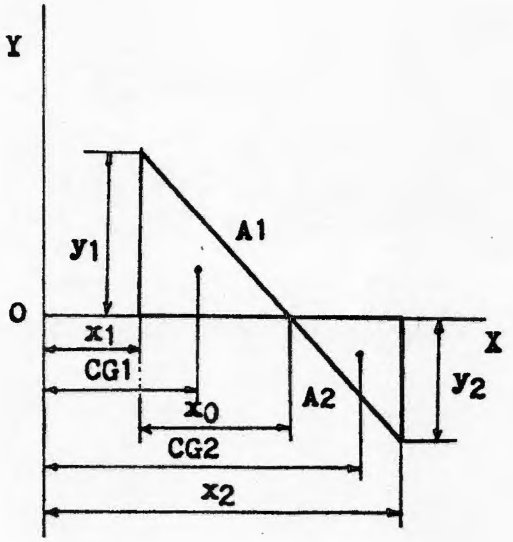


Figure 4.5 Case 3

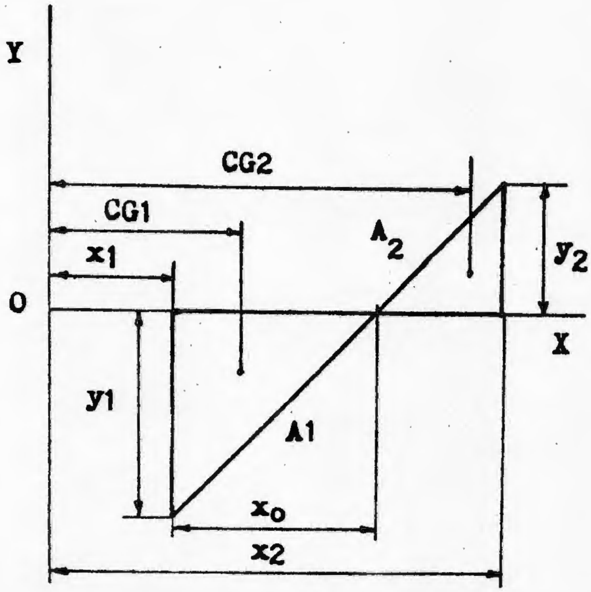


Figure 4.6 Case 4

$$A2 = Y_2(X_2 - X_1 - X_o)/2$$

Case 5. (see Figure 4.7)

Computations give same results as in Case 2.

Case 6. (see Figure 4.8)

Same results as in Case 1.

It is noted that the obtained results are the same

for cases 1 and 6,

for cases 2 and 5,

for cases 3 and 4.

The number of cases is then reduced from 6 to 3 as to formulas to be used.

C) Application to tube data.

A cross-sectional area containing the centers of drilled holes two feet from the edge of the tube is represented in Figure 4.10 (refer also to Figure 4.1). A sketch of a three-dimensional representation of the tube and an element of area of forces is shown in Figure 4.9.

The distance CGx computed in the six preceding cases is represented by the arc length AB in Figures 4.9 and 4.10.

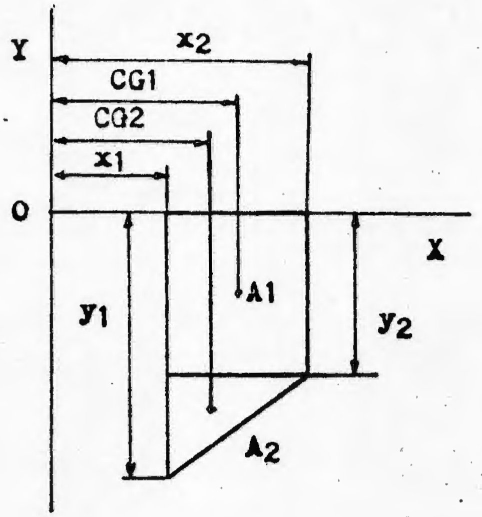
Lever arm D is:

$AC = r \sin \alpha$, with $\alpha = (CGX/r)$ radian

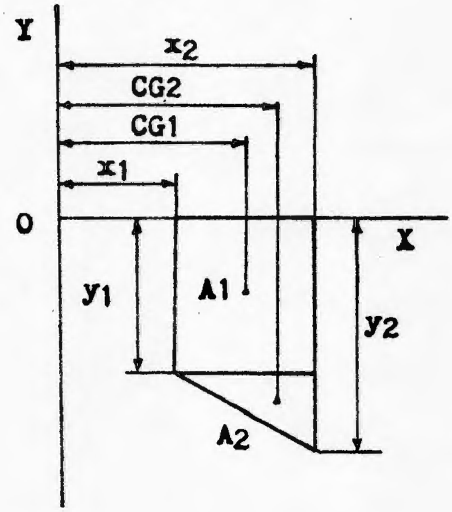
$$D = r \sin (CGx/r)$$

Therefore, a moment of an elementary area can be computed as follows:

$$\text{Moment} = \text{Force} * (\text{lever arm})$$



<u>Figure 4.7</u> Case 5



- <u>Figure 4.8</u> Case 6

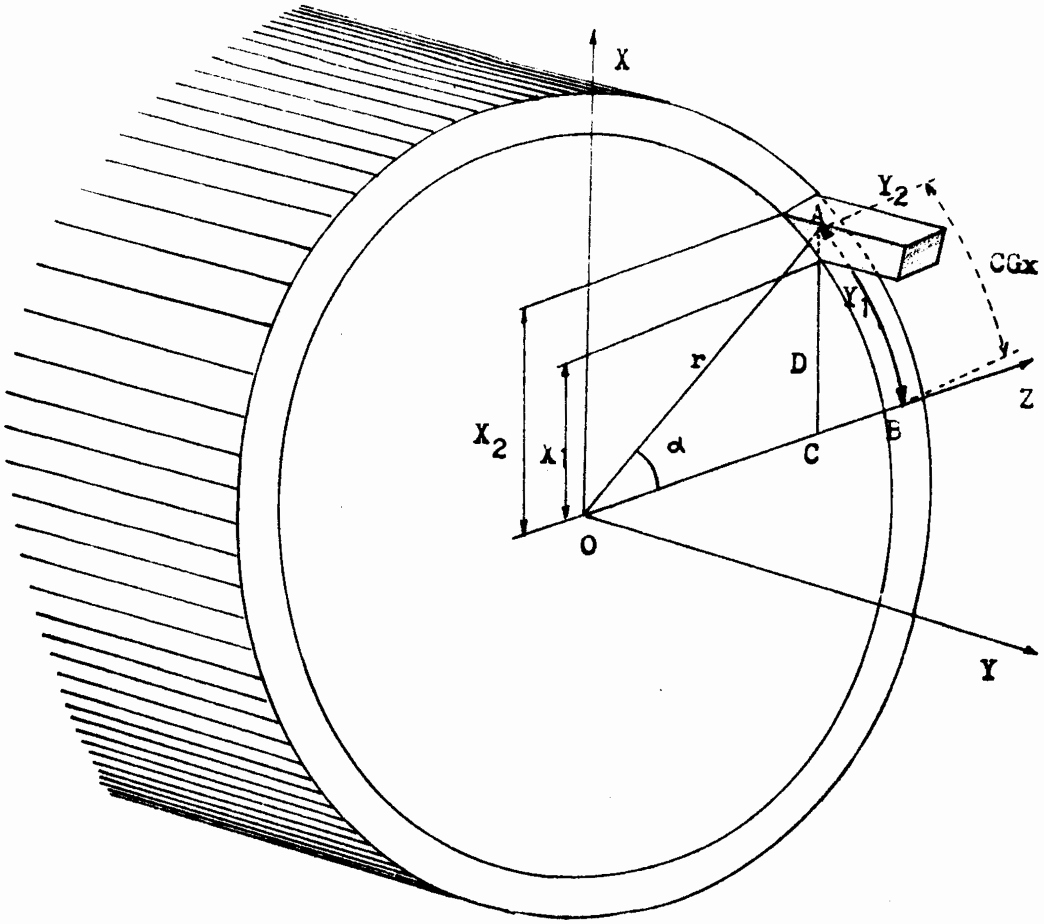


Figure 4.9 Sketch of a three-dimensional representation of the tube and an element of area of forces

Note. 1) For a true representation of an area of forces, cut the tube along longitudinal weld and flatten the cylinder on a plane parallel to XY plane.

2) The lever arm of the moment of forces with respect to the Z axis is $D = r \sin (CGx/r)$

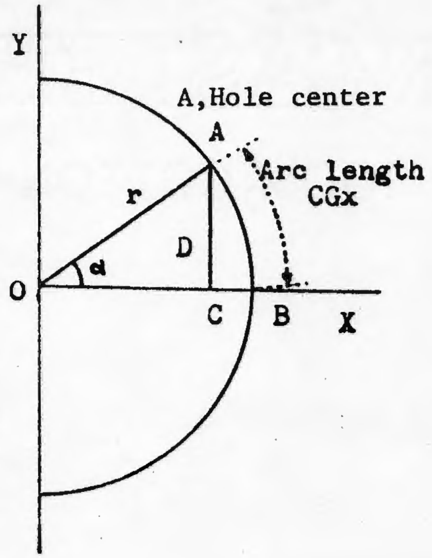


Figure 4.10 Cross-section of the tube

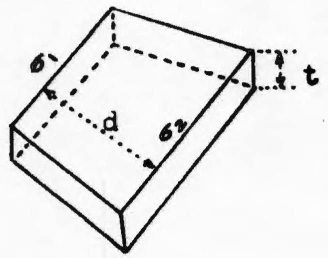


Figure 4.11 Sketch of a volume of forces

$$m = ((X_2 - X_1)Y_1)(r \sin \frac{X_2 + X_1}{2r}) +$$

$$((X_2 - X_1)(Y_2 - Y_1)/2)(r \sin \frac{X_2 + 2X_1}{3r})$$

This procedure is applied in the computer program.

D) Determination of dimensions and values for summation of forces and moments.

1) Dimensions and values for forces.

Suppose the area of forces is a trapezoid (see Figures 4.9 and 4.11).

Its area is:

Area = σ_L * d, with $\sigma_L = (\sigma_1 + \sigma_2)/2$

Volume of forces = Area * thickness

$= (\sigma_L\ d)\ t$

where:

d = distance or height of the trapezoid

t = thickness of the tube

r = radius of the tube

σ_L = average longitudinal residual stress

σ_y = yield stress of the steel

As σ_L/σ_y is used in ordinate and d/R used in abscissa in the curve (see Figures 3.27 and 4.1) the force is defined by:

$$\rho = \frac{\sigma_L}{\sigma_y} \times \frac{d}{r} \times \frac{t}{t}$$

$$\rho = \frac{\text{Force}}{\sigma_y\ r\ t}$$

let $P = \sigma_y\ r\ t$

As $\sigma_y = 40.9$ ksi

$r = 11"$

$t = 0.3125"$

$P = 140.6$ kip

Then, force is $\rho * (140.6)$ kip

Compute yield axial force P_y

$$P_y = \sigma_y * \frac{\pi}{4}(d^2 - d_1^2)$$

where $d = 22"$, $d_1 = 21.375"$, $P_y = 870.8$ kips

Suppose the ratio ρ calculated is 0.014, the summation of forces will be:

$$F = 0.014\ (140.6)\ \text{kips} = 1.968\ \text{kips}$$

And the ratio of summation of forces to yield axial load P_y is:

$$\frac{\Sigma F}{P_y} = \frac{1.9688}{870.8} = 0.0022$$

or

$$F = 0.0022\ P_y$$

2) Dimensions and value for moments.

Moment of forces = Volume of forces * (lever arm)

$$(\sigma_L)(d)(t)(\text{lever arm})$$

The moment is defined by

$$\rho_o = \frac{\sigma_L}{\sigma_y} \times \frac{d}{r} \times \frac{t}{t} \times \frac{(1.a)}{r}$$

$$\rho_o = \frac{\text{Moment}}{\sigma_y\ r^2 t}$$

$$\text{Moment} = \rho_o(\sigma_y\ r^2 t)$$

with

$$\sigma_y \, r^2 t = (40.9)(11)^2(0.3125)$$

$$= 1546.5 \text{ kip.in.}$$

Then

$$\text{Moment} = \rho_o * (1546.5) \text{ kip.in.}$$

Suppose

$$\rho_o = 0.15$$

$$M = 0.15 * (1546.5) = 231.98 \text{ kip.in.}$$

Compute Moment at first yield,

$$M_y = s.F_y$$

As $s = \frac{\pi}{4}d^2t$ (approximate formula for the section modulus)

where d = 22", t = 0.3125"

Then,

$$M_y = (40.9)(\pi/4 * 22^2 * 0.3125)$$

$$= 4859 \text{ kip.in.}$$

Ratio of summation of moments to moment at first yield:

$$\frac{\Sigma M}{My} = \frac{231.98}{4859} = 0.047$$

or

$$\Sigma M = 0.047 \, M_y$$

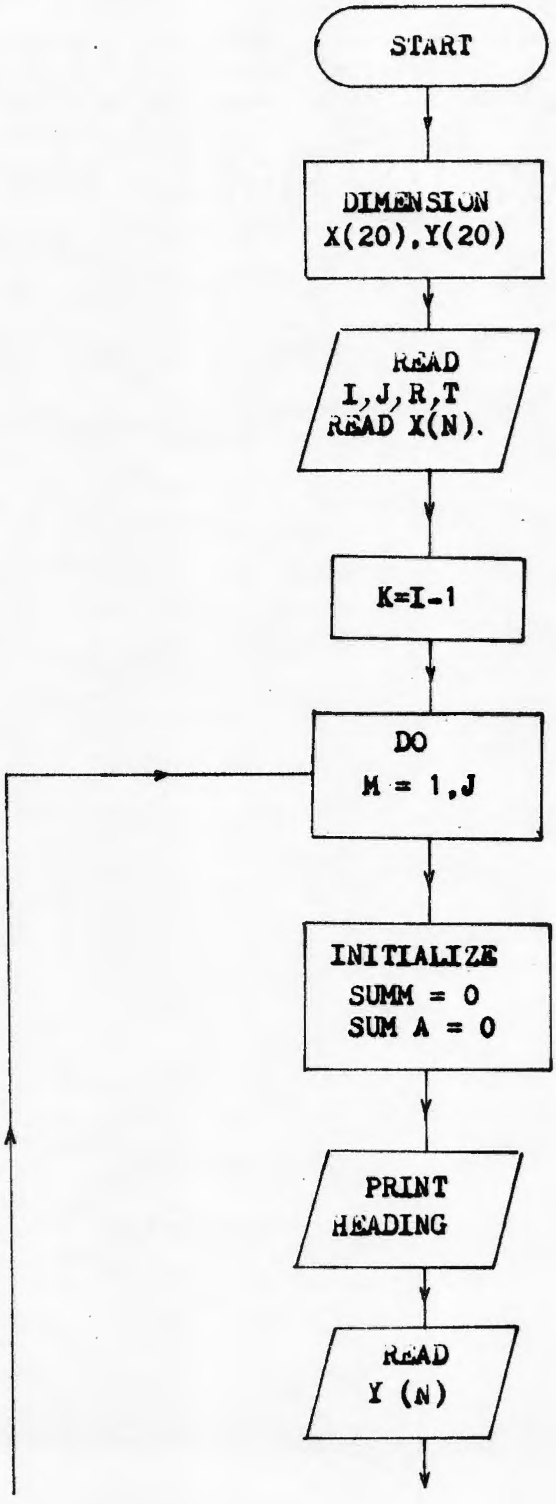
4.1.2 Flow chart.

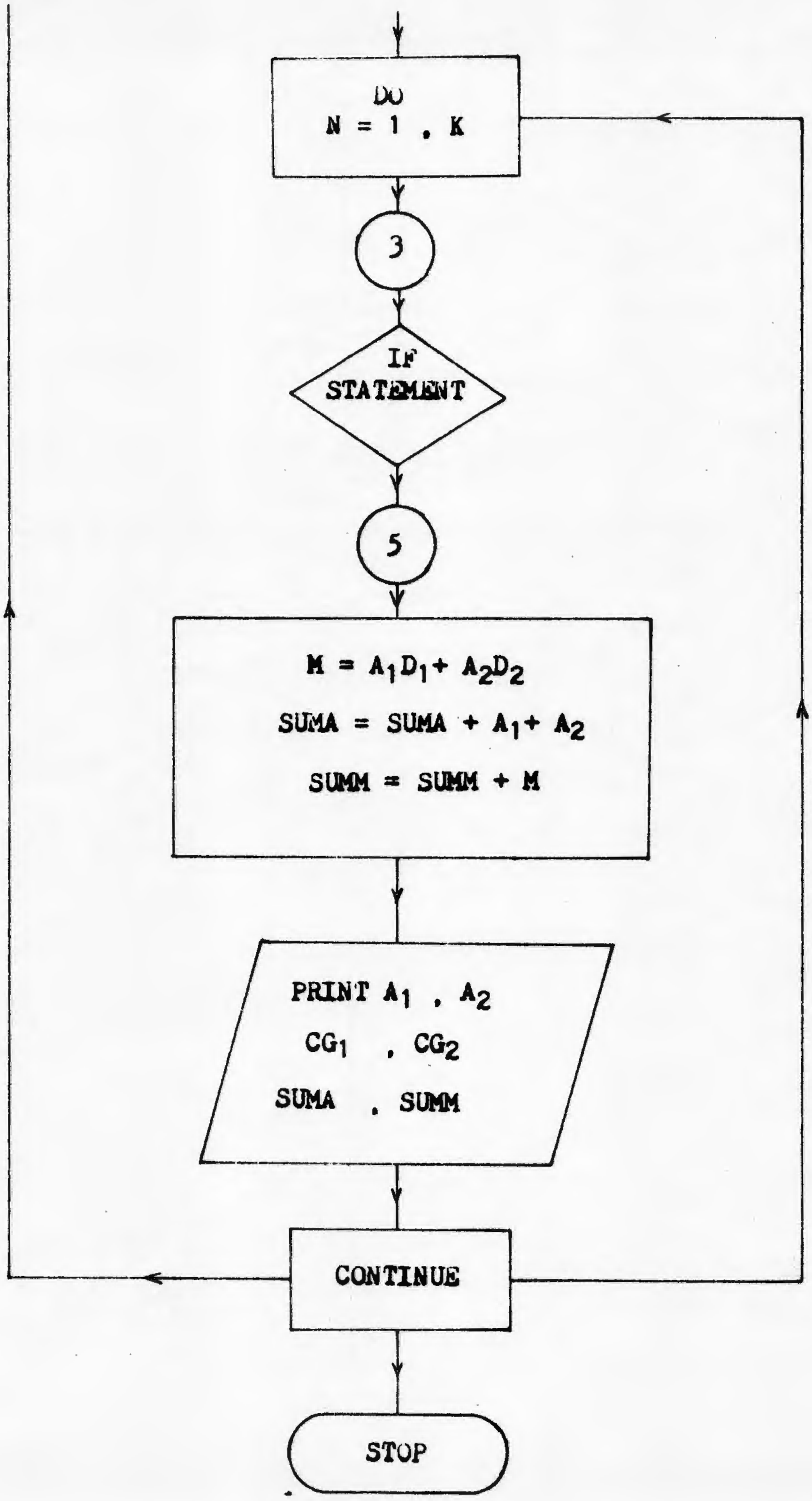
A flow diagram for the computer program is shown in Figure 4.12.

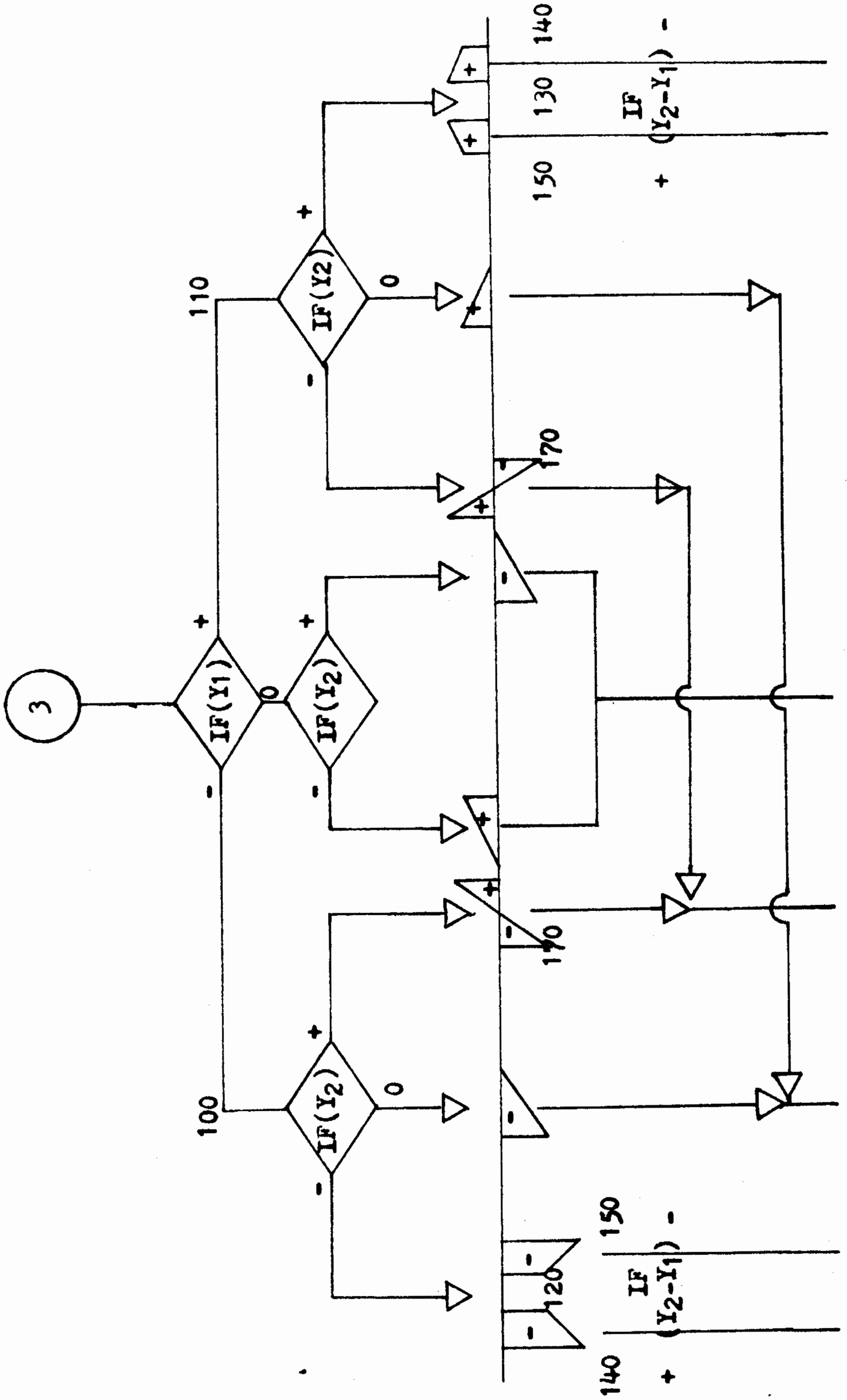
4.1.3 Computer program.

A computer program for calculating the summation of forces and

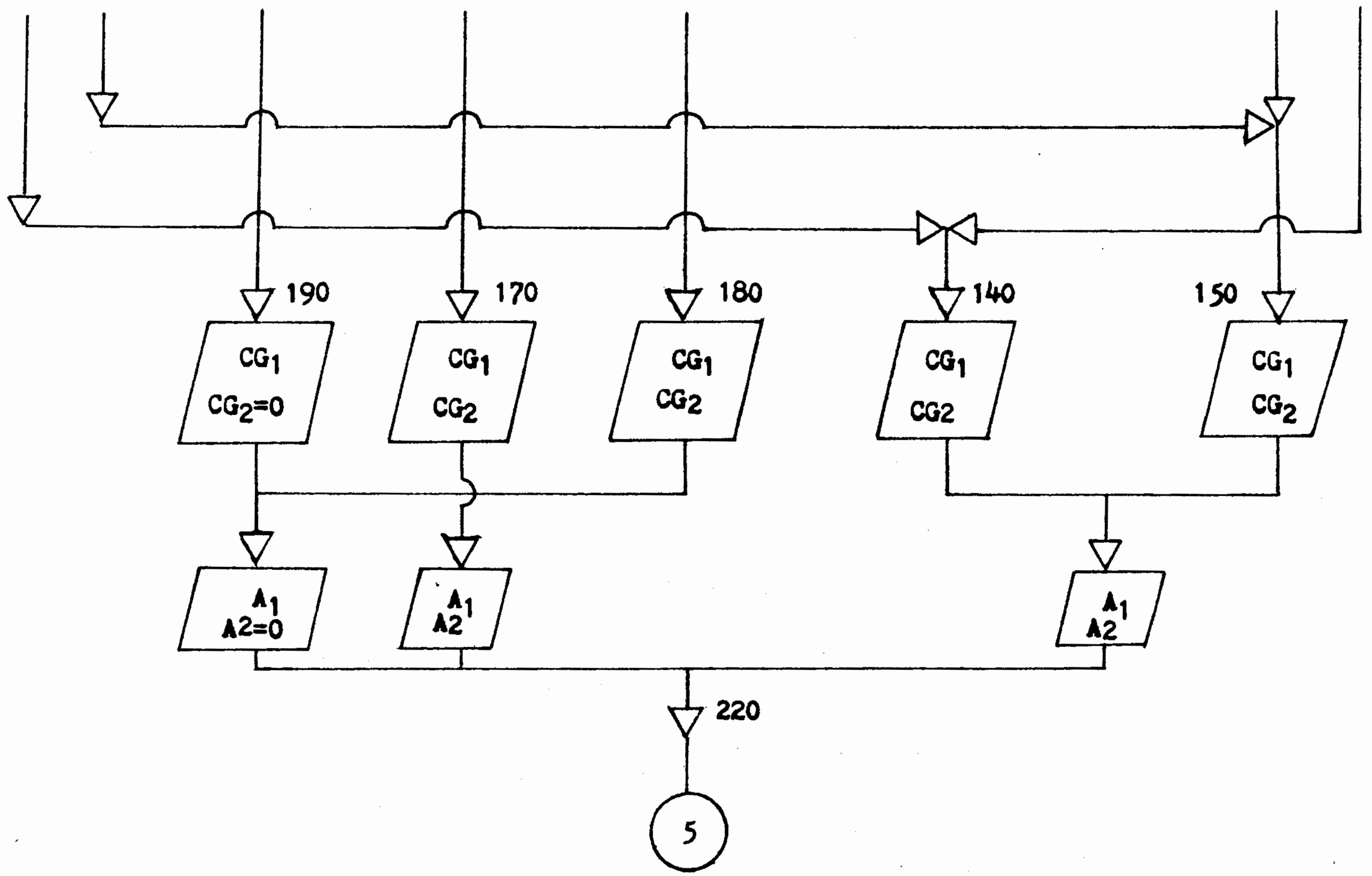
Figure 4.12 Flow diagram




DO
N = 1 , K
3
IF
STATEMENT
5
M = A1D1+ A2D2
SUMA = SUMA + A1+ A2
SUMM = SUMM + M
PRINT A1 , A2
CG1 , CG2
SUMA , SUMM
CONTINUE
STOP



3
IF(Y1)
IF(Y_2)
IF(Y2)
IF(Y_2)
100
110
120
130
140
150
170
IF (Y_2-Y_1)
IF (Y_2-Y_1)


190
170
180
140
150
CG_1
$CG_2=0$
CG_1
CG_2
CG_1
CG_2
CG_1
CG2
CG_1
CG_2
A_1
A2=0
A_1
A2
A_1
A2
220
5

summation of moments relating to measured longitudinal residual stresses derived from measured strains is presented in Appendix IX.

It is noted that a preparation is required for converting raw data to input as the bending moment of the system of longitudinal residual stresses is taken about the Z axis (see Figure 4.1). This preparation useful for a future research presented in detail for case 1 in Appendix XI with the statement and figure defining the area of forces limited by the curve and the Z axis.

4.1.4 Results of the computer program.

A) Results relating to stresses obtained from measured strains with variable 4A and 4B, Table 3.11, case 1.

The output obtained from the compilation of the computer program and data file shown in Appendix IX and relating to the initial longitudinal residual stresses derived from measured strains with variable 4A and 4B, case 1, Table 3.11, results in:

Summation of forces above the Z axis: -0.06 8125

Summation of forces below the Z axis: 0.0100453

Total summation of forces: -0.06 8125 + 0.100453 = 0.032328

This result is a dimensionless number. As described in section 4.1.1.d, it must be multiplied by 140.6 kip to yield the force to find.

$$\Sigma F = 0.032328\ (140.6) = 4.545 \text{ kip}$$

Expressed in percent of P_y, the summation of forces is:

$$4.545(100)/870.8 = 0.5\% \text{ of } P_y$$

-Summation of moments: -0.092374 - 0.06026 = -0.152634

This summation of moments must be multiplied by 1546.5 kip.in. to yield inch-kips, thus:

-0.152634 (1546.5) kip.in. = -236.05 kip.in.

Expressed in percent of M_y, the summation of moments is:

-236.05 (100)/4859 = -5% of M_y.

Computer output and data file are given in Appenxid IX.

B) Results related to stresses obtained from measured strains but computed with theoretical 4A and 4B, Table 3.11, case 2.

Summation of forces above the Z axis: -0.061359

Summation of forces below the Z axis: 0.098718

Total summation of forces: -0.061359 + 0.098718 = 0.037359

Total summation of moments:

-0.086489 - 0.057948 = -0.144437

The same approach as the above one gives the final results:

Summation of forces: 0.006 P_y

Summation of moments: -0.046 M_y.

Computer output and data file are given in Appendix X.

C) Results related to stresses obtained from the strain-relaxation method with 4A and 4B, Table 3.11, case 3.

Summation of forces above the Z axis: -0.104375

Summation of forces below the Z axis: 0.085563

Total summation of forces:

-0.104375 + 0.085563 = -0.018812

Total summation of moments:

-0.088679 - 0.049973 = -0.13864

The same approach as the above one gives the final results:

Summation of forces = $-0.003\ P_y$

Summation of moments: $-0.044\ M_y$

Computer output and data file are given in Appendix X.

D) Results related to stress obtained from the strain-separation method with 4A and 4B, Table 3.11, case 4.

Summation of forces above the Z axis: -0.043141

Summation of forces below the Z axis: 0.055731

Total summation of forces:

$$-0.043141 + 0.055731 = 0.01264$$

Total summation of moments:

$$-0.055679 - 0.031260 = -0.086939$$

The same approach as the above gives the final results:

Summation of forces: $0.002\ P_y$

Summation of moments: $-0.027\ M_y$

Computer output and data file are given in Appendix X.

These results are summarized in Table 4.1.

4.2 EVALUATION OF RESULTS

In light of the data in Table 4.1 it is seen that the results of case 1 are marginally acceptable giving an unbalance of 10% M_y. Cases 2 and 3 have slightly better rotational equilibrium with the smallest unbalance being 8.8% M_y. Case 4 gives the best results with 0.4% P_y translational and 5.4% M_y rotational unbalance for the whole tube. This is to be expected since the calibration coefficients for case 4 were found using a more refined technique than any of the other cases. The unbalance present in case 4 could be attributed to the choice of sector division required for data generation for the computer program.

TABLE 4.1

RESULTS ON FORCE AND MOMENT BALANCE

Case No.	Method	Calibration Coefficients (10^{-8})	ΣF	ΣM	
				Half Section	Whole Section
1	Measurement	Variable	0.005 P_y	-0.05 M_y	10% M_y
2	Computation of theoretical calibration coefficients	4A=-0.86 4B=-2.41	0.006 P_y	-0.046 M_y	9.2% M_y
3	Method of strain-relaxation	4A=-0.93 4B=-2.08	-0.003 P_y	-0.044 M_y	8.8% M_y
4	Method of strain separation	4A=-1.30 4B=-2.23	0.002 P_y	-0.027 M_y	5.4% M_y

CHAPTER V

SUMMARY, CONCLUSION AND RECOMMENDATIONS

The experiment of drilling outside and inside holes in a welded fabricated steel tube, according to the hole-drilling technique has been carried out, as was the experiment of tensioning a steel plate specimen for calibration. Longitudinal residual stresses in the tube have been obtained from the hole drilling experiments. Calibration coefficients used to convert data from the hole drilling experiment to longitudinal stresses were obtained in two ways 1) Calibration coefficients were derived analytically by theory and using those recommended by published literature, 2) Calibration coefficients have also been computed from calibration operations performed according to the method of strain relaxation due to drilling and the method of strain separation. Calibration coefficients have been used in adjusting the data of the tube. Balance of forces and moments provides a comparison of the results of measurements and the quality of the results of refinement.

Results will be summarized, conclusion and recommendations will be presented in the following sections.

5.1 SUMMARY OF RESULTS

The experiments of outside and inside hole drilling on the steel tube and the calibration experiments conducted on the test plate followed by the balancing operations of forces and moments have yielded the following results:

A) The measured strains on the tube have shown that no further strains were recorded after the values of the hole depth reached the hole diameter value.

B) The strains measured for outside and inside holes and the corresponding longitudinal residual stresses computed have suggested that the stress field in the tube was well behaved through its thickness.

C) The field of longitudinal residual stresses in the tube assumed to be uniform through its thickness, has been substantiated by a numerical calculation using two factors of proportionality derived from the test plate in the calibration experiment and applied to the tube.

D) Four different kinds of calibration coefficients were used, they are summarized as follows:

1. Variable. $4A = -0.83$ to $-1.05(10^{-8})$, $4B = -2.37$ to $-2.68(10^{-8})$
2. $4A = -0.86(10^{-8})$, $4B = -2.41(10^{-8})$
3. $4A = -0.93(10^{-8})$, $4B = -2.08(10^{-8})$
4. $4A = -1.30(10^{-8})$, $4B = -2.23(10^{-8})$

As a comparison of the accuracy of the results, a statical equilibrium test in translation and rotation was performed for each of the residual stress distribution obtained using the above four kinds of calibration coefficients. The results are summarized in Table 4.1.

E) Finally, a curve plotting the most acceptable values of residual stresses based on equilibrium of forces and moments is repeated in Figure 5.1.

5.2 CONCLUSION

The results in the preceding section show the adequacy of the

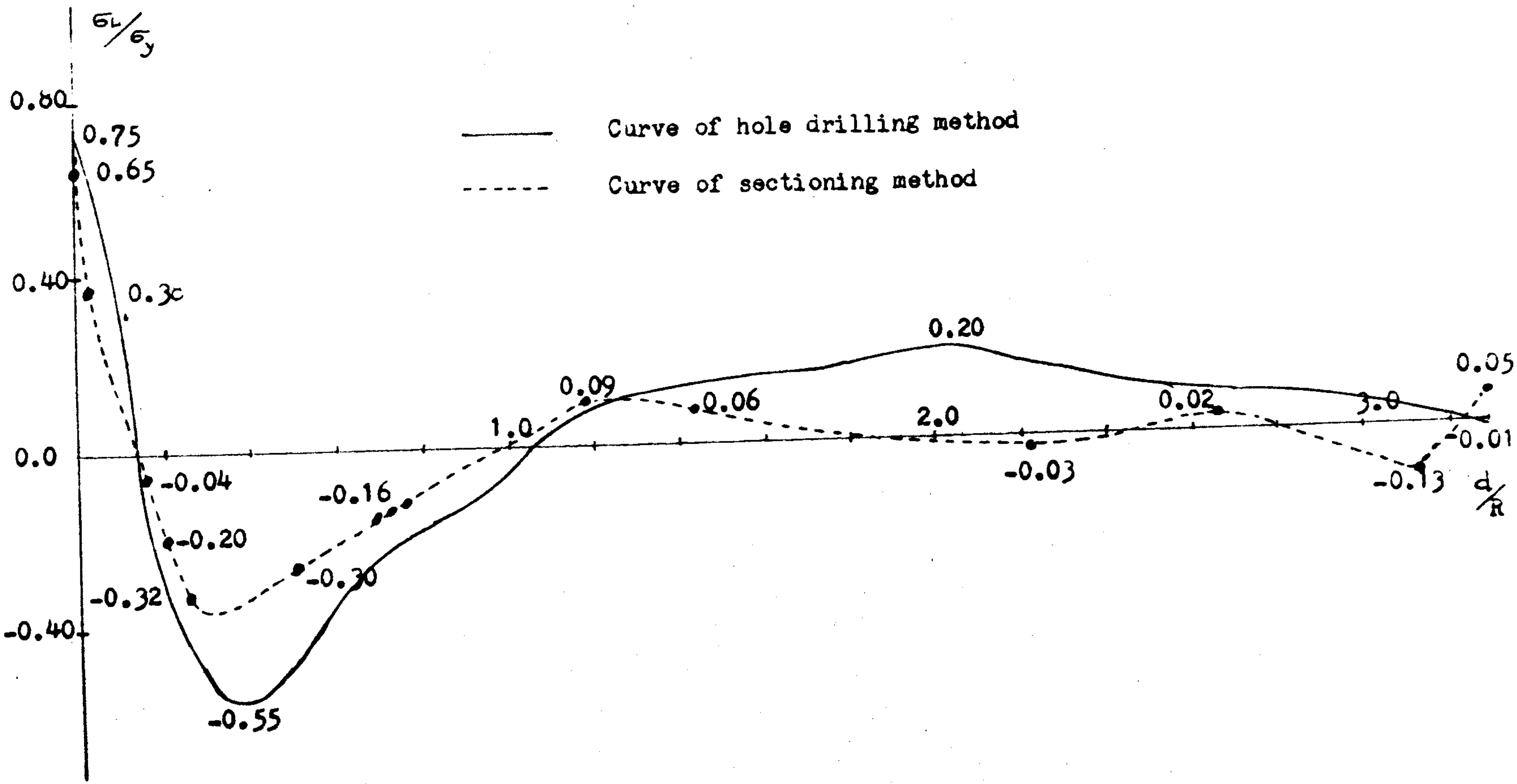


Figure 5.1 Curve of residual stresses by hole drilling method (solid line) and curve by method of sectioning (dashed line)

hole-drilling technique in the measurement of residual stresses, provided the hole drilling experiments are conducted in conjunction with the calibration carried out preferably according to strain separation method and using material similar to the main experiment material.

For comparison, the curve of the longitudinal residual stresses found by the slicing method by others, is compared to the results of this investigation in Figure 5.1.

5.3 RECOMMENDATIONS

1) Translational and rotational equilibrium calculations confirmed the good results of the hole-drilling strain-gage experiments conducted on the steel tube. The curve of longitudinal residual stresses shown by a solid line in Figure 5.1 compared to results by others, further confirms the quality of the hole-drilling technique in measuring residual stresses. Our first recommendation is that calibration coefficients should always be used to refine the curve of stresses found and the method of strain separation would be the most efficient calibration method.

2) The solid curve shown in Figure 5.1 characterizes the distribution of residual stresses in the steel tube. For a more practical purpose, a simplified curve comprising three straight solid lines shown in Figure 5.2 is recommended. The balance of forces and moments derived from the pertinent curve is obtained with a high precision (summation of forces: $-0.0005\ P_y$, summation of moments: $0.00009\ M_y$).

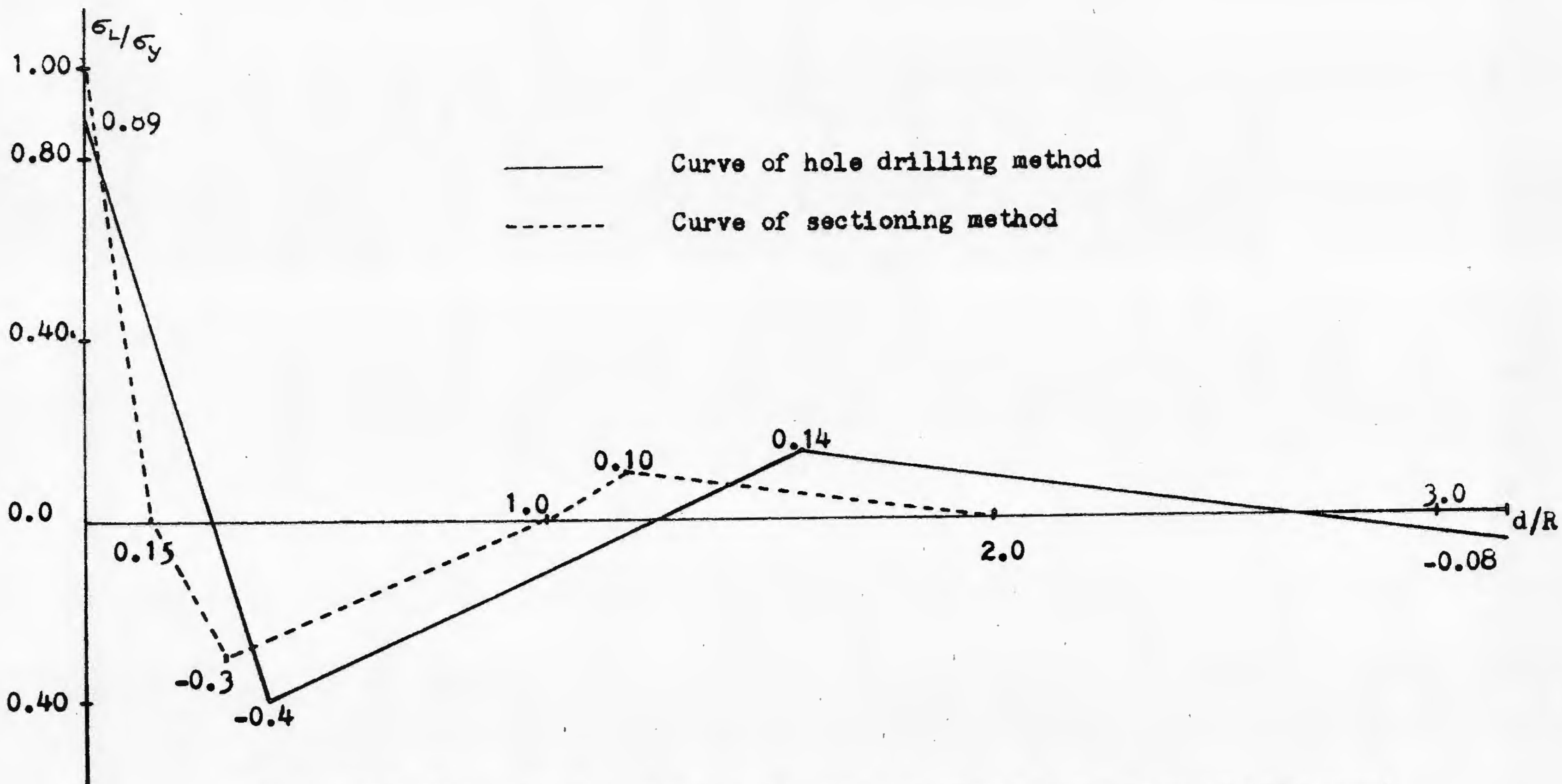
For comparison, a dotted straight line curve proposed by

Ross and Chen (5) is shown in the same Figure 5.2. This curve is a simplified alternative of Ross and Chen presented in Figure 5.1. A computer program applied to this simplified curve with the same conditions as for the simplified curve of our tube, shows that the precision obtained in balancing forces and moments was equivalent:

Summation of forces: -0.00012 P_y (against -0.0005 P_y for our curve)

Summation of moments: 0.0026 M_y (against 0.00009 M_y for our curve)

Finally, our simplified straight line approximation presented in Figure 5.2 to represent longitudinal residual stresses in the steel tube, is recommended for use in further studies of the effects of residual stresses on failure loads.



<u>Figure 5.2</u> Ross and Chen simplified curve by sectioning method (dotted line) and simplified curve by hole-drilling method (solid line) to represent residual stresses.

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APPENDIX I

DERIVATION OF EQUATIONS (3)

(3)

$$\sigma_1 = \frac{(A + B \cos 2\beta)\, \varepsilon_a - (A - B \cos 2\beta)\, \varepsilon_c}{4AB \cos 2\beta}$$

$$\sigma_2 = \frac{(A + B \cos 2\beta)\, \varepsilon_c - (A - B \cos 2\beta)\, \varepsilon_a}{4AB \cos 2\beta}$$

$$\tan 2\beta = \frac{\varepsilon_a - 2\varepsilon_b + \varepsilon_c}{\varepsilon_a - \varepsilon_c}$$

In section 2.2.2, the following expression has been established =

$$\varepsilon_r = [A + B \cos 2\alpha]\ \sigma_1 + [A + B \cos 2(\alpha + 90)]\ \sigma_2$$

Extending to directions a and c, and replacing α by β, it becomes =

(2a)

$$\varepsilon_a = [A + B \cos 2\beta]\ \sigma_1 + [A + B \cos 2(\beta + 90)]\ \sigma_2$$

$$\varepsilon_c = [A + B \cos 2(\beta + 90)]\ \sigma_1 + [A + B \cos 2(\beta + 180)]\ \sigma_2$$

Noting that =

$$\cos 2(\beta + 90) = \cos (2\beta + 180) = -\cos 2\beta$$

$$\cos 2(\beta + 180) = \cos (2\beta + 360) = \cos 2\beta$$

(2a) can be rewritten =

$$[A + B \cos 2\beta]\ \sigma_1 + [A - B \cos 2\beta]\ \sigma_2 = \varepsilon_a$$

(2b)

$$[A - B \cos 2\beta]\ \sigma_1 + [A + B \cos 2\beta]\ \sigma_2 = \varepsilon_c$$

Dividing both members by the coefficients of σ_1 =

$$\sigma_1 + \frac{A - B \cos 2\beta}{A + B \cos 2\beta}\ \sigma_2 = \frac{\varepsilon_a}{A + B \cos 2\beta}$$

$$\sigma_1 + \frac{A + B \cos 2\beta}{A - B \cos 2\beta}\ \sigma_2 = \frac{\varepsilon_c}{A - B \cos 2\beta}$$

Using direct elimination (subtract member to member) =

$$\left(\frac{A - B \cos 2\beta}{A + B \cos 2\beta} - \frac{A + B \cos 2\beta}{A - B \cos 2\beta}\right) \sigma_2 = \frac{\varepsilon_a}{A + B \cos 2\beta} - \frac{\varepsilon_c}{A - B \cos 2\beta}$$

Developing and rearranging terms =

$$[(A - B \cos 2\beta)^2 - (A + B \cos 2\beta)^2\ \sigma_2 = \varepsilon_a(A - B \cos 2\beta) - \varepsilon_c(A + B \cos 2\beta)$$

$$[-2\ AB \cos 2\beta - 2\ AB \cos 2\beta]\ \sigma_2 = \varepsilon_a(A - B \cos 2\beta) - \varepsilon_c(A + B \cos 2\beta)$$

Finally:

$$\sigma_2 = \frac{(A + B \cos 2\beta)\ \varepsilon_c - (A - B \cos 2\beta)\ \varepsilon_a}{4\ AB \cos 2\beta}$$

With the same approach =

$$\sigma_1 = \frac{(A + B \cos 2\beta)\ \varepsilon_a - (A - B \cos 2\beta)\ \varepsilon_c}{4\ AB \cos 2\beta}$$

—

Proof of =

$$\tan 2\beta = \frac{\varepsilon_a - 2\varepsilon_b + \varepsilon_c}{\varepsilon_a - \varepsilon_c}$$

Remind =

(2b)

$$\varepsilon_a = [A + B \cos 2\beta]\ \sigma_1 + [A - B \cos 2\beta]\ \sigma_2$$

$$\varepsilon_c = [A - B \cos 2\beta]\ \sigma_1 + [A + B \cos 2\beta]\ \sigma_2$$

then = (2c)

$$\varepsilon_a + \varepsilon_c = 2\ A(\sigma_1 + \sigma_2)$$

$$\varepsilon_a - \varepsilon_c = 2\ B \cos 2\beta(\sigma_1 - \sigma_2)$$

therefore =

$$\cos 2\beta = \frac{\varepsilon_a - \varepsilon_c}{2\ B\ (\sigma_1 - \sigma_2)}$$

Calculate sin 2β.

For direction b =

$$\varepsilon_b = [A + B \cos 2(\beta + 45°)]\ \sigma_1 + [A + B \cos 2(\beta + 135°)]\ \sigma_2$$

$$\varepsilon_b = [A - B \sin 2\beta]\ \sigma_1 + [A + B \sin 2\beta]\ \sigma_2$$

$$\varepsilon_b = A\ (\sigma_1 + \sigma_2) + B\ (\sigma_2 - \sigma_1) \sin 2\beta$$

Therefore =

$$\sin 2\beta = \frac{A\ (\sigma_1 + \sigma_2) - \varepsilon_b}{B\ (\sigma_2 - \sigma_1)}$$

Compute tan 2β:

$$\tan 2\beta = \frac{\sin 2\beta}{\cos 2\beta} = \frac{A(\sigma_1 + \sigma_2) - \varepsilon_b}{B(\sigma_2 - \sigma_1)} \times \frac{2B(\sigma_1 - \sigma_2)}{\varepsilon_a - \varepsilon_c}$$

$$= \frac{2A(\sigma_1 + \sigma_2) - 2\varepsilon_b}{\varepsilon_a - \varepsilon_c}$$

As $2A(\sigma_1 + \sigma_2) = \varepsilon_a + \varepsilon_c$, from (2c)

Finally:

$$\tan 2\beta = \frac{\varepsilon_a + \varepsilon_c - \varepsilon_{eb}}{\varepsilon_a - \varepsilon_c}$$

$$= \frac{\varepsilon_a - 2\varepsilon_b + \varepsilon_c}{\varepsilon_a - \varepsilon_c}$$

Developing and rearranging terms =

$$(A - B\cos 2\beta)(A - B\sin 2\beta) - (A + B\sin 2\beta)(A + B\cos 2\beta)\quad \sigma_2 =$$

$$\varepsilon_a\,(A - B\sin 2\beta) - \varepsilon_b\,(A + B\cos 2\beta)$$

Developing first member =

$$A^2 - AB\sin 2\beta - AB\cos 2\beta + B^2\sin 2\beta\cos 2\beta$$

$$-(A^2 + AB\cos 2\beta + AB\sin 2\beta + B^2\sin 2\beta\cos 2\beta) =$$

$$-2AB\,(\sin 2\beta + \cos 2\beta)$$

Then =

$$\sigma_2 = \frac{(A + B\cos 2\beta)\,\varepsilon_b - (A - B\sin 2\beta)\,\varepsilon_a}{2AB\,(\sin 2\beta + \cos 2\beta)}$$

With the same approach =

$$\sigma_1 = \frac{(A + B\sin 2\beta)\,\varepsilon_a - (A - B\cos 2\beta)\,\varepsilon_b}{2AB\,(\sin 2\beta + \cos 2\beta)}$$

APPENDIX III

DERIVATION OF EQUATIONS (3a)

(3a)

$$\sigma_1 = \frac{S}{4A} + \frac{D}{4B \cos 2\beta}$$

$$\sigma_2 = \frac{S}{4A} - \frac{D}{4B \cos 2\beta}$$

with $\tan 2\beta = \dfrac{S - 2\varepsilon_b}{D}$

$S = \varepsilon_a + \varepsilon_c$

$D = \varepsilon_a - \varepsilon_c$

Rewrite equation (3) from Appendix I

(3)

$$\sigma_1 = \frac{(A + B \cos 2\beta)\ \varepsilon_a - (A - B \cos 2\beta)\ \varepsilon_c}{4AB \cos 2\beta}$$

$$\sigma 2 = \frac{(A + B \cos 2\beta)\ \varepsilon_c - (A - B \cos 2\beta)\ \varepsilon_a}{4AB \cos 2\beta}$$

Developing the second member and grouping terms =

$$\sigma_1 = \frac{A\ (\varepsilon_a - \varepsilon_c) + B \cos 2\beta\ (\varepsilon_a + \varepsilon_c)}{4AB \cos 2\beta}$$

$$= \frac{\varepsilon_a + \varepsilon_c}{4A} + \frac{\varepsilon_a - \varepsilon_c}{4B \cos 2\beta}$$

$$\sigma_1 = \frac{S}{4A} + \frac{D}{4B \cos 2\beta}$$

With the same approach =

$$\sigma_2 = \frac{S}{4A} - \frac{D}{4B \cos 2\beta}$$

Coefficients 4A and 4B can be computed by adding or subtracting member to member, from equations (3a)

$$4A = \frac{2S}{\sigma_1 + \sigma_2}$$

$$4B = \frac{2D}{(\sigma_1 - \sigma_2) \cos 2\beta}$$

APPENDIX IV

COMPUTER PROGRAM FOR CALCULATING PRINCIPAL STRESSES σ_1 & σ_2 WHEN STRAINS ε_A , ε_B , ε_C ARE KNOWN

```
ED DOCK

        READ(7,4)EA,EB,EC
   4    FORMAT(3E7.0)
        S=EA+EC
        D=EA-EC
        WRITE(18,6)S,D
     6  FORMAT(5X'SUM S='E15.6,5X'DIFF D='E15.6)
        READ(7,8)F4A,F4B
   8    FORMAT(2E7.2)
        C=ATAN((S-2*EB)/D)
         C1=C*180./3.14159
         WRITE(18,10)C1
     10  FORMAT(5X'ANGLE 2B='F8.2)
         BCOS=COS(C)
         S1=S/(F4A)+D/(F4B*BCOS)
         S2=S/(F4A)-D/(F4B*BCOS)
         WRITE(18,20)S1,S2
     20  FORMAT(3X'STRESS 1='F11.2,3X'STRESS 2='F11.2)
         STOP
         END
EOF..
EOT..
```

DATA FILE AND OUTPUT

```
ED DOC3

1 +004E-6+027E-6+075E-6
2 -132E-8-220E-8
3 EOF
EOT..

FR 5,6
FO.WNME,DOCK
AS 18=*3
AS 7=DOC3
VX.N

    SUM S=   0.790000E-04   ,  DIFF D=  -0.710000E-04
    ANGLE 2B=  -19.40
  STRESS 1=   -2563.36  , STRESS 2=   -9406.34
STOP
```

APPENDIX V

Graphical method using a stress MOHR's CIRCLE to determine longitudinal and circumferential stresses σ_L & σ_T when principal stresses σ_1 & σ_2 are known.

Build the stress Mohr's circle with diameter AB, abscissa OA = σ_1 and abscissa OB = σ_2.

The direction β of gage a with respect to σ_1 is given by formula :

$$\tan 2\beta = \frac{S - 2\varepsilon_b}{D}$$

If the direction b coincides with the transverse axis of the pipe, and α is the angle of the longitudinal axis with direction σ_1:

$$\alpha = 90° - (45° + \beta) = 45° - \beta$$

In the Mohr's circle, the angle used for a direction is double: (see Fig. A.6.1)

$$2\alpha = 90° - 2\beta$$

Knowing the direction of σ_1 on the Mohr's circle (radius wA), open an angle AwC = $2\alpha = 90° - 2\beta$, the abscissa of M, intersection with the circle is the longitudinal stress σ_L.

The abscissa of Point N diametrically opposite to M is the transverse or circumferential stress σ_T.

<u>Application</u>. Hole 1.

$\varepsilon_a = 4, \quad \varepsilon_b = 27, \quad \varepsilon_c = 75$

$\sigma_1 = -6070$ psi, $\sigma_2 = -12091$ psi

$2\beta = 19° \; 22 \qquad \beta = 9° \; 41$

—

$\alpha = 90° - (45 + 9.41) = 35° \; 19$

$2\alpha = 70° \; 38$

Build a Mohr's circle with =

OB = -12091 psi

OA = -6070 psi

From wA, open an angle AwC = $2\alpha = 70° \; 38$; its second side cuts the circle in points M & N.

Point M, $\sigma_L = -8080$ psi

Point N, $\sigma_T = -10090$ psi.

APPENDIX VI

COMPUTER PROGRAM

ANALYTICAL STUDY

Determination of longitudinal stress σ_L and circumferential stress σ_T when principal stresses σ_1 and σ_2 are known.

Part I. Longitudinal Stress σ_L.

Equation of a Mohr's circle of stress, when principal stresses σ_1, σ_2 are known:

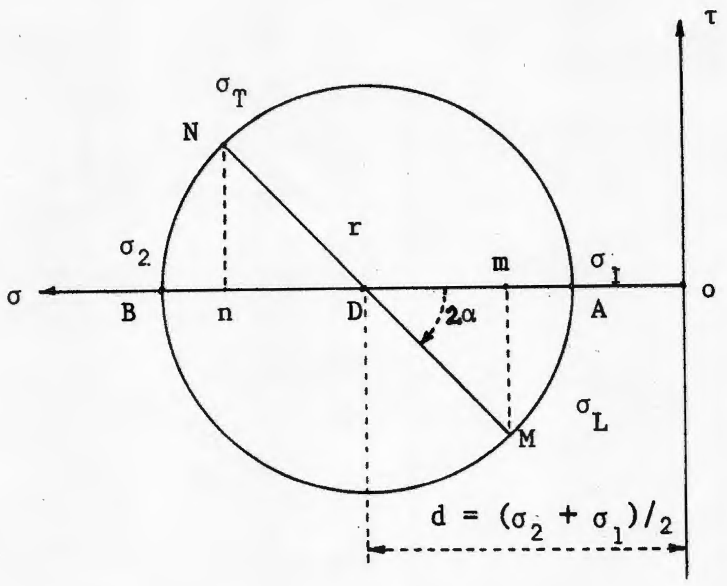


Figure A.6.1. Stress Mohr's circle.

(1) $$(X - OD)^2 + Y^2 = r^2$$

As $OD = d = \dfrac{\sigma_2 - \sigma_1}{2}$

$$r = \frac{\sigma_2 - \sigma_1}{2}$$

According to equation (2a) (Appendix 1) =

(2a) $$\varepsilon_a + \varepsilon_c = 2A\ (\sigma_1 + \sigma_2)$$

$$\varepsilon_a - \varepsilon_c = 2B \cos 2\beta\ (\sigma_1 - \sigma_2)$$

(2a) can be rewritten =

$$\sigma_1 + \sigma_2 = \frac{\varepsilon_a + \varepsilon_c}{2A} = \frac{S}{2A}$$

$$\sigma_1 - \sigma_2 = \frac{\varepsilon_a - \varepsilon_c}{2B \cos 2\beta} = \frac{D}{2B \cos 2\beta}$$

Therefore =

$$OD = \frac{\sigma_2 + \sigma_1}{2} = \frac{S}{4A}$$

$$R = \frac{\sigma_2 - \sigma_1}{2} = \frac{-D}{2B \cos 2\beta}$$

After substitution of terms, equation (1) becomes:

$$(X - \frac{S}{4A})^2 + Y^2 = \frac{D^2}{(4B \cos 2\beta)^2}$$

To find the longitudinal stress, an angle 2α is built from DA, its second side cuts the Mohr's circle at point M. The abscissa of point M is the longitudinal stress.

Equation of the Straight Line DM.

a) Its slope is tan 2α, (α being the angle of the longitudinal stress) with direction σ_1.

As shown in Appendix V:

$$\alpha = 45° - \beta$$

$$2\alpha = 90° - 2\beta$$

$$\tan 2\alpha = \tan (90° - 2\beta)$$

$$= \cot\text{an}\ 2\beta$$

$$= \frac{1}{\tan 2\beta}$$

According to equation (3), Appendix I,

$$\tan 2\beta = \frac{S - 2\varepsilon_b}{D}$$

$$\text{Let } M = \frac{S - 2\varepsilon_b}{D}$$

Then

$$\tan 2\alpha = \frac{1}{\tan 2\beta} = \frac{D}{S - 2\varepsilon_b} = \frac{1}{M}$$

b) As the straight line passes through point D (abcissa =

$\frac{\sigma_2 + \sigma_1}{2} = \frac{S}{4A}$, ordinate = 0)

$$Y = \frac{1}{M} X + N \ , \quad M = \frac{S - 2\varepsilon_b}{D}$$

$$0 = \left(\frac{D}{S - 2\varepsilon_b}\right) \frac{S}{4A} + N$$

$$N = \frac{-SD}{4A\ (S - 2\varepsilon_b)}$$

$$Y = \left(\frac{D}{S - 2\varepsilon_b}\right) X - \frac{SD}{4A\ (S - 2\varepsilon_b)}$$

$$Y = \frac{1}{M} X - \frac{S}{4A} \times \frac{1}{M}$$

c) Abscissa of point M. intersection of DM and Mohr's circle =

Line DM =

$$Y = \frac{1}{M} (X - \frac{S}{4A})$$

Mohr's circle =

$$(X - \frac{S}{4A})^2 + Y^2 = (\frac{D}{4B \cos 2\beta})^2$$

Let $E = \frac{S}{4A}$,

Line DM =

(1a) $$Y = \frac{1}{M} (X - E)$$

Mohr's circle =

(1b) $$(X - E)^2 + Y^2 = (\frac{D}{4B \cos 2\beta})^2$$

As M is common to DM and and circle, its coordinates satisfy both equations (1a), (1b).

Raise (1a) to square and substitute into (1b) =

$$Y^2 = \frac{1}{M^2} (X - E)^2$$

$$(X - E)^2 + \frac{1}{M^2} (X - E)^2 = (\frac{D}{4B \cos 2\beta})^2$$

$$(1 + \frac{1}{M^2}) (X - E)^2 - (\frac{D}{4B \cos 2\beta})^2 = 0$$

$$(\frac{M^2 + 1}{M^2}) (X - E)^2 - (\frac{D}{4B \cos 2\beta})^2 = 0$$

$$(X - E)^2 - \left(\frac{M^2}{1 + M^2}\right) \left(\frac{D}{4B \cos 2\beta}\right)^2 = 0$$

Developing and rearranging terms:

$$X^2 - 2EX + E^2 - \left(\frac{M^2}{1 + M^2}\right) \left(\frac{D}{4B \cos 2\beta}\right)^2 = 0$$

Quadratic equation in X with =

$$A = 1$$
$$B = -2E$$
$$C = E^2 - \left(\frac{M^2}{1 + M^2}\right) \left(\frac{D}{4B \cos 2\beta}\right)^2$$

<u>Existence of Roots</u>:

Discriminant $\Delta' = B'^2 - AC$

$$= E^2 - \left[E^2 - \left(\frac{M^2}{1 + M^2}\right) \left(\frac{D}{4B \cos 2\beta}\right)^2\right]$$

$$= +\left(\frac{M^2}{1 + M^2}\right) \left(\frac{D}{4B \cos 2\beta}\right)^2 > 0$$

Therefore, the equation will always have roots.

—

<u>Part II. Circumferential Stress σ_T</u>.

σ_T is obtained with point N diametrically opposite to M (σ_L).

COMPUTER PROGRAM

for longitudinal and circumferential stresses

```
ED SIGMA6

         READ(7,4)EA,EB,EC
    4    FORMAT(3E7.0)
         S=EA+EC
         D=EA-EC
         WRITE(18,6)EA,EB,EC
    6    FORMAT(5X'EA='E15.4,5X'EB='E15.4,5X'EC='E15.4)
         READ(7,8)F4A,F4B
    8    FORMAT(2E7.2)
         WRITE(18,9)F4A,F4B
     9    FORMAT(5X'F4A='E15.4,5X'F4B='E15.4)
          Q=S/(F4A)
          XM=(S-2.*EB)/D
            XL=ATAN(XM)
          XL1=XL*180./3.14159
          WRITE(18,10)XL1
    10    FORMAT(5X'ANGLE XL1='F8.2)
          BCOS=COS(XL)
C         COMPUTE STRESS 1 AND STRESS 2
          S1=S/(F4A)+D/(F4B*BCOS)
          S2=S/(F4A)-D/(F4B*BCOS)
          WRITE(18,12)S1,S2
    12    FORMAT(3X'STRESS 1='F11.2,3X'STRESS 2='F11.2)
C         COMPUTE'LONGITUDINAL STRESS'
          A=1
          B=-2.*Q
          C=Q**2-(XM**2/(1.+XM*XM))*(D/((F4B)*BCOS))**2
          TEMP=B**2-4.*A*C
          IF(S1)30,30,60
```

```
   30 IF(S1-S2)40,40,50
   40 IF(XL1)41,41,46
   41 YL=(-B-(TEMP)**0.5)/(2.*A)
   46 YL=(-B+(TEMP)**0.5)/(2.*A)
   50 IF(XL1)51,51,56
   51 YL=(-B+(TEMP)**0.5)/(2.*A)
   56 YL=(-B-(TEMP)**0.5)/(2.*A)
   60 IF(S1-S2)70,70,80
   70 IF(XL1)71,71,76
   71 YL=(-B-(TEMP)**0.5)/(2.*A)
   76 YL=(-B+(TEMP)**0.5)/(2.*A)
   80 IF(XL1)81,81,86
   81 YL=(-B+(TEMP)**0.5)/(2.*A)
   86 YL=(-B-(TEMP)**0.5)/(2.*A)
      WRITE(18,90)YL
   90 FORMAT(2X'LONGITUDINAL STRESS='E15.4)
C     COMPUTE 'CIRCUMFERENTIAL STRESS'
      IF(S1)130,130,160
  130 IF(S1-S2)140,140,150
  140 IF(XL1)141,141,146
  141 YT=(-B+(TEMP)**0.5)/(2.*A)
  146 YT=(-B-(TEMP)**0.5)/(2.*A)
  150 IF(XL1)151,151,156
  151 YT=(-B-(TEMP)**0.5)/(2.*A)
  156 YT=(-B+(TEMP)**0.5)/(2.*A)
  160 IF(S1-S2)170,170,180
  170 IF(XL1)171,171,176
  171 YT=(-B+(TEMP)**0.5)/(2.*A)
  176 YT=(-B-(TEMP)**0.5)/(2.*A)
  180 IF(XL1)181,181,186
  181 YT=(-B-(TEMP)**0.5)/(2.*A)
  186 YT=(-B+(TEMP)**.5)/(2.*A)
      WRITE(18,200)YT
  200 FORMAT(2X'CIRCUMFERENTIAL STRESS='E15.4)
      STOP
      END
EOT..
```

DATA FILE AND OUTPUT

for longitudinal and circumferential stresses

```
ED DOCK3

1 +004E-6+027E-6+075E-6
2 -087E-8-250E-8
3 EOF
EOT..
```

```
FO.WNME,SIGMA6
AS 18=*3
AS 7=DOCK3
VX.N

    EA=     0.4000E-05     EB=     0.2700E-04     EC=     0.7500E-04
    F4A=    -0.8700E-08     F4B=    -0.2500E-07
    ANGLE XL1=  -19.40
  STRESS 1=   -6069.55   STRESS 2=  -12091.37
 LONGITUDINAL STRESS=    -0.1008E+05
 CIRCUMFERENTIAL STRESS=    -0.8080E+04
STOP
```

APPENDIX VII

Graphical method to determine longitudinal and circumferential strains ε_L & ε_T, when strains ε_a, ε_b, ε_c are known.

A. Construct Mohr's circle.

Build a system of orthogonal coordinates:

-vertical axis or axis of half shear-strains.

-horizontal axis or axis of strains (see Figure A7.1)

On the axis of strains, plot:

$OA = \varepsilon_a$

$OB = \varepsilon_b$

$OC = \varepsilon_c$

Mark point D middle of AC. On the verticals of A and C, build:

$$Aa = BD \quad , \quad Cc = BD$$

aD and Dc are the radii of Mohr's circle.

On the vertical of B, build:

$$Bb = DA \quad \text{or} \quad Bb = DC$$

Db is also the radius of the circle. The three right angles aAD, cCD and bBD are equal.

Draw the Mohr's circle passing on a, b, c.

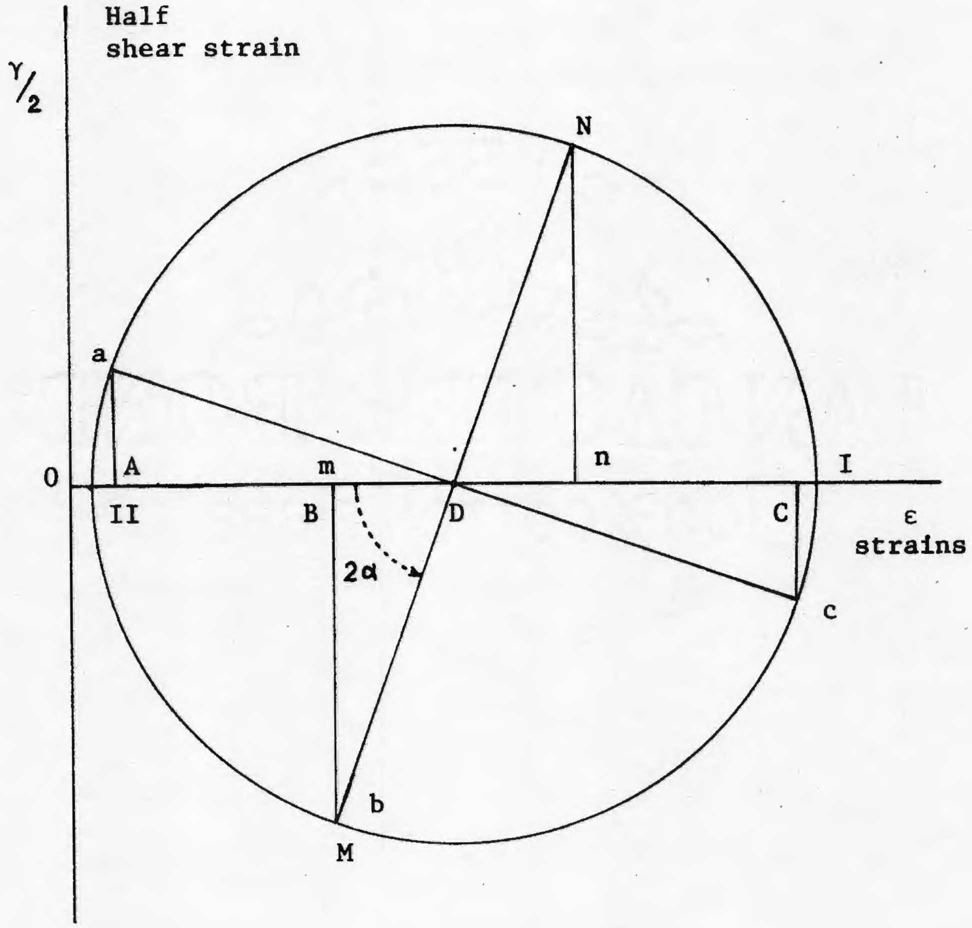


Figure A.7.1. Strain Mohr's circle

Note the location of points I and II giving the maximum and minimum strains ε_1 and ε_2.

B. Check location of directions (gages) a, b, c.

As the three directions a, b, c are 45° apart and counterclockwise, the three points a, b, c on the Mohr's circle 90° apart (double angle in Mohr's circle) and counterclockwise are good ones.

C. Compute the longitudinal strain.

1.) In Appendix V =

$$\alpha = 45° - \beta$$

and

$$\tan 2\beta = \frac{S - 2\varepsilon_b}{D} \qquad \text{(Appendix I)}$$

The angle of principal axis ε_1 with ε_L is 2α, in Mohr's circle =

$$2\alpha = 90° - 2\beta$$

The second side of the angle 2α built from the principal strain direction cuts the Mohr's circle at point M, its abscissa is the longitudinal strain.

2.) Numerical example.

Case of Hole 1 .

Strains = $\varepsilon_a = 4 \times 10^{-6}$, $\varepsilon_b = 27 \times 10^{-6}$, $\varepsilon_c = 75 \times 10^{-6}$.

Plots points A, B, C on strain axis:

$$OA = 4 \quad , \quad OB = 27 \quad , \quad OC = 75$$

Mark point D, middle of AC.

Build Aa = Cc = BD = 13 .

Draw the strain Mohr's circle from D with radius OD = 37.5.

Compute $\tan 2\beta = \dfrac{4 - 2(27) + 75}{4-75}$

$$= -\frac{25}{71}$$

$2\beta = -19°\ 24^{mi}$, $\beta = -9°\ 24^{mi}$

$2\alpha = 90° - 2\beta = 70°\ 18^{mi}$

From the principal axis, open an angle $2\alpha = 70°\ 18^{mi}$, its second side cuts the Mohr's circle, first at point M then, at point N.

Longitudinal strain om = 27×10^{-6}.

Circumferential strain on = 52×10^{-6}.

APPENDIX VIII

COMPUTER PROGRAM

VIII A. ANALYTICAL APPROACH

A. General Considerations

The construction of a strain Mohr's circle has been shown in Appendix VII.

The problem consists of determining the directions and magnitudes of the longitudinal and circumferential strains when the angle α of the direction a with the longitudinal direction is known (β = angle of direction a with principal axis).

Let:

$$\delta = \alpha - \beta$$

In the strain Mohr's circle, double angles are considered =

$$2\delta = 2\alpha - 2\beta$$

As tan 2β can be determined by equation (3) and tan 2α is known, 2δ can be calculated.

$$\tan 2\delta = \tan (2\alpha - 2\beta)$$

$$\tan 2\delta = \frac{\tan 2\alpha - \tan 2\beta}{1 + \tan 2\alpha \tan 2\beta} \qquad (1)$$

B. Equation of Strain Mohr's Circle (See Fig. A7.1)

In strain Mohr's circle, are known =

$$\varepsilon_a = OA$$
$$\varepsilon_b = OB$$
$$\varepsilon_c = OC$$

For convenience's sake, let OA = a
OB = b
OC = c

Then $OC = d = \dfrac{a + c}{2}$

Radius of Mohr's circle =

$$R^2 = \overline{BD}^2 + \overline{Bb}^2$$

As
$$Bb = DC = c - d$$
$$= c - \frac{a + c}{2}$$
$$Bb = \frac{c - a}{2}$$
$$BD = d - b = \frac{a + c}{2} - b = \frac{a + c - 2b}{2}$$

Let
$$S = a + c$$
$$D = a - c$$

$$R^2 = \left(\frac{c - a}{2}\right)^2 + \left(\frac{a + c - 2b}{2}\right)^2$$
$$= \left(\frac{D}{2}\right)^2 + \left(\frac{S - 2b}{2}\right)^2$$

Equation of the strain Mohr's circle =

$$\left(X - \frac{a + c}{2}\right)^2 + Y^2 = \frac{1}{4}\left[D^2 + (S - 2b)^2\right]$$

Let $Q = S - 2b$

$$(X - \frac{S}{2})^2 + Y^2 = \frac{1}{4} (D^2 + Q^2)$$

C. Equation of the Straight Line DM.

$$Y = MX + N$$

If 2δ is the angle of the principal direction and the longitudinal direction, its second side cuts Mohr's circle at point M, its abscissa is the value of the longitudinal strain.

The slope of DM is $\tan 2\delta$ which can be computed by the equation (1)

$$Y = \tan 2\delta \; X + N$$

As the line DM passes through point D $(\frac{a + c}{2}, 0)$, the coordinates of D satisfy this equation: (See Figure A.7.1)

$$0 = \tan 2\delta \left(\frac{S}{2} \right) + N$$

$$N = - \frac{S}{2} \tan 2\delta$$

Then:

$$Y = \tan 2\delta \left(X - \frac{S}{2}\right)$$

D. Magnitude of Longitudinal Strain

As M is common to the line and the circle, its coordinates satisfy both equations.:

(1a) Line: $Y = \tan 2\delta \left(X - \frac{S}{2}\right)$

(1b) Circle: $(X - \frac{S}{2})^2 + Y^2 = \frac{1}{4}(D^2 + Q^2)$

Raise (1a) to square and substitute into (1b):

$$Y^2 = \tan 2\delta \, (X - \frac{S}{2})$$

$$(X - \frac{S}{2})^2 + \tan^2 2\delta \, (X - \frac{S}{2})^2 = \frac{1}{4} (D^2 + Q^2)$$

or

$$(1 + \tan^2 2\delta)(X - \frac{S}{2})^2 - \frac{1}{4} (D^2 + Q^2) = 0$$

or

$$(X - \frac{S}{2})^2 - \frac{(D^2 + Q^2)}{4(1 + \tan^2 2\delta)} = 0$$

Developing and rearranging terms =

$$X^2 - SX + \frac{S^2}{4} - \frac{(D^2 + Q^2)}{4(1 + \tan^2 2\delta)} = 0$$

Quadratic equation in X where =

$$A = 1$$

$$B = -S$$

$$C = \frac{S^2}{4} - \frac{(D^2 + Q^2)}{4(1 + \tan^2 2\delta)}$$

<u>Discussion</u>

a) Condition of root existence:

$$\Delta = B^2 - 4AC \geq 0.$$

b) When angle $2\delta < 90°$, the longitudinal strain equals:

$$\varepsilon_L = X_1 = \frac{-B - \sqrt{B^2 - 4\,AC}}{2A}$$

and the circumferential strain equals:

$$\varepsilon_T = X_2 = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

VIII B

COMPUTER PROGRAM

for longitudinal and circumferential strains.

```
ED EPSIL3

         READ(7,4)EA,EB,EC
     4   FORMAT(3F7.0)
         S=EA+EC
         D=EA-EC
         Q=EA+EC-2.*EB
         WRITE(18,6)S,D,Q
    6    FORMAT(2X'SUM S='E15.4,2X,'DIFF D='E15.4,2X,'SUM-2EB='E15.4)
         READ(7,7)F2A
    7    FORMAT(F6.2)
          T=TAN(F2A)
          R=(S-2.*EB)/D
          F2B=ATAN(R)
          F2B1=F2B*180./3.14159
          F2D=F2A-F2B1
          WRITE(18,8)F2D
     8    FORMAT(2X'ANGLE 2D='F6.2)
          Y=(T-R)/(1.+T*R)
```

```
      A=1.
      B=-S
      C=(S**2)/4.-(D**2+Q**2)/(4.*(1.+(Y)**2))
      WRITE(18,9)A,B,C
    9 FORMAT(4X'A='E15.4,4X'B='E15.4,4X'C='E15.4)
      TEMP=B**2-4.*A*C
      XM=Y
      IF(TEMP)300,10,10
   10 IF(XM)80,70,60
C     COMPUTE"LONGITUDINAL STRAIN"
   60 ZL=(-B+(TEMP)**0.5)/(2.*A)
   70 ZL=-B/(2.*A)
   80 ZL=(-B-(TEMP)**0.5)/(2.*A)
      WRITE(18,90)ZL
   90 FORMAT(2X"LONGITUDINAL STRAIN="E15.4)
C     COMPUTE"CIRCUMFERENTIAL STRAIN"
      IF(XM)180,170,160
  160  ZT=(-B-(TEMP)**0.5)/(2.*A)
  170  ZT=-B/(2.*A)
  180 ZT=(-B+(TEMP)**0.5)/(2.*A)
      WRITE(18,190)ZT
  190 FORMAT(2X'CIRCUMFERENTIAL STRAIN='E15.4)
  300 STOP
      END
EOT..
```

VIII C

DATA FILE AND OUTPUT

for longitudinal and circumferential strains.

```
ED APE1

1 +004E-6+027E-6+075E-6
2 109.20
3 EOF
EOT..


FR 5,6
FO.WNME,EPSIL3
AS 18=*3
AS 7=APE1
VX.N


 SUM S=      0.7900E-04  DIFF D=     -0.7100E-04  SUM-2EB=      0.2500E-04
 ANGLE 2D=128.60
   A=     0.1000E+01    B=     -0.7900E-04     C=      0.3763E-09
 LONGITUDINAL STRAIN=      0.5092E-05
 CIRCUMFERENTIAL STRAIN=      0.7391E-04
STOP
```

APPENDIX IX

IX A. COMPUTER PROGRAM
for force and moment balance.

```
ED RETR1

      DIMENSION X(20),Y(20)
      READ(12,10)I,J,R,T,DC
   10 FORMAT(2I3,3F12.4)
      READ(12,20)(X(N),N=1,I)
   20 FORMAT(F11.3)
      WRITE(18,30)
      K=I-1
      DO 240 M=1,J
      SUMM=0.0
       SUMA=0.0
    30 FORMAT(2X'  AREA1    LEVER ARM      AREA 2       LEVER ARM   SUM FORCES
      1         SUM MOMENTS')
       READ(12,40)(Y(N),N=1,I)
    40 FORMAT(F11.3)
       DO 240 N=1,K
       IF(Y(N))100,90,110
    90 IF(Y(N+1))180,95,180
    95 A1=0.0
       A2=0.0
       CG1=0.0
       CG2=0.0
       GO TO 220
   100 IF(Y(N+1))120,190,170
```

```
110   IF(Y(N+1))170,190,130
120   IF(Y(N+1)-Y(N))150,135,140
130   IF(Y(N+1)-Y(N))140,135,150
135   A1=(X(N+1)-X(N))*Y(N)
      CG1=(X(N+1)+X(N))/2.0
      A2=0.0
      CG2=0.0
      GO TO 220
140   CG1=(X(N+1)+X(N))/2.0
      CG2=(X(N+1)+2.0*X(N))/3.0
      A1=(X(N+1)-X(N))*Y(N+1)
      A2=(X(N+1)-X(N))*(Y(N)-Y(N+1))/2.0
      GO TO 220
150   CG1=(X(N+1)+X(N))/2.0
      CG2=(X(N+1)*2.0+X(N))/3.0
      A1=(X(N+1)-X(N))*Y(N)
      A2=(X(N+1)-X(N))*(Y(N+1)-Y(N))/2.0
      GO TO 220
170   XO=(X(N+1)-X(N))*(-Y(N))/(Y(N+1)-Y(N))
      CG1=(XO/3.0)+X(N)
      CG2=(2.0*X(N+1)+X(N)+XO)/3.0
      A1=(XO*Y(N))/2.0
      A2=(X(N+1)-(X(N)+XO))*Y(N+1)/2.0
      GO TO 220
180   CG1=(2.0*X(N+1)+X(N))/3.0
      CG2=0.0
      A1=(X(N+1)-X(N))*Y(N+1)/2.0
      A2=0.0
      GO TO 220
190   CG1=(X(N+1)+2.0*X(N))/3.0
      CG2=0.0
      A1=(X(N+1)-X(N))*Y(N)/2.0
      A2=0.0
```

```
 220  A1=A1*T
      A2=A2*T
      IF(CG1)222,221,222
 221  D1=0.0
      GO TO 223
 222  D1=R*SIN(CG1/R)-DC
 223  IF(CG2)225,224,225
 224  D2=0.0
      GO TO 226
 225  D2=R*SIN(CG2/R)-DC
 226  XM=A1*D1+A2*D2
      SUMA=SUMA+A1+A2
      SUMM=SUMM+XM
      WRITE(18,230)A1,D1,A2,D2,SUMA,SUMM
 230  FORMAT(6E13.6)
 240  CONTINUE
      READ(12,2)I,J,R,T,DC
   2  FORMAT(2I3,3F12.4)
      READ(12,4)(X(N),N=1,I)
   4  FORMAT(F11.3)
      K=I-1
      DO 340 M=1,J
      SUMM=0.0
      SUMA=0.0
      WRITE(18,6)
   6  FORMAT(2X' AERA 1    LEVER ARM     AREA 2       LEVER ARM   SUM FORCES
     1          SUM MOMENTS')
      READ(12,8)(Y(N),N=1,I)
   8  FORMAT(F11.3)
      DO 340 N=1,K
      IF(Y(N))200,12,210
  12  IF(Y(N+1))280,14,280
  14  A1=0.0
```

```
      A2=0.0
      CG1=0.0
      CG2=0.0
      GO TO 320
200   IF(Y(N+1))218,290,270
210   IF(Y(N+1))270,290,232
218   IF(Y(N+1)-Y(N))250,235,241
232   IF(Y(N+1)-Y(N))241,235,250
235   A1=(X(N+1)-X(N))*Y(N)
      CG1=(X(N+1)+X(N))/2.0
      A2=0.0
      CG2=0.0
      GO TO 320
241   CG1=(X(N+1)+X(N))/2.0
      CG2=(X(N+1)+2.0*X(N))/3.0
      A1=(X(N+1)-X(N))*Y(N+1)
      A2=(X(N+1)-X(N))*(Y(N)-Y(N+1))/2.0
      GO TO 320
250   CG1=(X(N+1)+X(N))/2.0
      CG2=(X(N+1)*2.0+X(N))/3.0
      A1=(X(N+1)-X(N))*Y(N)
      A2=(X(N+1)-X(N))*(Y(N+1)-Y(N))/2.0
      GO TO 320
270   X0=(X(N+1)-X(N))*(-Y(N))/(Y(N+1)-Y(N))
      CG1=(X0/3.0)+X(N)
      CG2=(2.0*X(N+1)+X(N)+X0)/3.0
      A1=(X0*Y(N))/2.0
      A2=(X(N+1)-(X(N)+X0))*Y(N+1)/2.0
      GO TO 320
280   CG1=(2.0*X(N+1)+X(N))/3.0
      CG2=0.0
      A1=(X(N+1)-X(N))*Y(N+1)/2.0
```

```
      A2=0.0
      GO TO 320
290   CG1=(X(N+1)+2.0*X(N))/3.0
      CG2=0.0
      A1=(X(N+1)-X(N))*Y(N)/2.0
      A2=0.0
320   A1=A1*T
      A2=A2*T
      IF(CG1)322,321,322
321   D1=0.0
      GO TO 323
322   D1=R*SIN(CG1/R)-DC
323   IF(CG2)325,324,325
324   D2=0.0
      GO TO 326
325   D2=R*SIN(CG2/R)-DC
326   XM=-A1*D1-A2*D2
      SUMA=SUMA+A1+A2
      SUMM=SUMM+XM
      WRITE(18,330)A1,D1,A2,D2,SUMA,SUMM
330   FORMAT(6E13.6)
340   CONTINUE
      STOP
      END
EOT..
```

IX B. DATA FILE AND OUTPUT

```
ED REFILE13

18   1     11.0          0.3125
 0.0
 0.27
 0.4
 0.51
 0.56
 0.67
 0.785
 0.89
  0.98
  1.09
  1.18
  1.29
  1.38
  1.41
  1.45
  1.47
  1.51
  1.57
  0.26
  0.18
  0.13
  0.03
  0.0
 -0.16
 -0.24
 -0.32
 -0.42
 -0.66
 -0.75
 -0.66
  -0.45
 -0.3
  0.0
  0.2
  0.5
  0.8
 6   1     11.0          0.3125
 0.0
 0.39
 0.78
 1.18
 1.31
 1.57
 0.26
 0.34
 0.19
 0.14
 0.12
 0.02
```

```
FO.WNME,RETR1
AS 18=*3
AS 12=REFILE13
VX.N

   AREA1     LEVER ARM      AREA 2        LEVER ARM   SUM FOR          SUM MOMENTS
0.151875E-01 0.134997E+00 0.337500E-02 0.899990E-01 0.185625E-01 0.235401E-02
0.528125E-02 0.334948E+00 0.101563E-02 0.313291E+00 0.248594E-01 0.444114E-02
0.103125E-02 0.454870E+00 0.171875E-02 0.436552E+00 0.276094E-01 0.566055E-02
0.234375E-03 0.526465E+00 0.000000E+01 0.000000E+01 0.278437E-01 0.578394E-02
0.275000E-02 0.632983E+00 0.000000E+01 0.000000E+01 0.250937E-01 0.404323E-02
0.575000E-02 0.726970E+00-0.143750E-02 0.746093E+00 0.179062E-01-0.120935E-02
0.787500E-02 0.836691E+00-0.131250E-02 0.854139E+00 0.871875E-02-0.891935E-02
0.900000E-02 0.933875E+00-0.140625E-02 0.948819E+00-0.168750E-02-0.186585E-01
0.144375E-01 0.103347E+01-0.412500E-02 0.105172E+01-0.202500E-01-0.379176E-01
0.185625E-01 0.113299E+01-0.126562E-02 0.114791E+01-0.400781E-01-0.604015E-01
0.226875E-01 0.123241E+01-0.154688E-02 0.121419E+01-0.643125E-01-0.902400E-01
0.126563E-01 0.133173E+01-0.295313E-02 0.131683E+01-0.799219E-01-0.110983E+00
0.281250E-02 0.139126E+01-0.703125E-03 0.138630E+01-0.834375E-01-0.115871E+00
0.187500E-02 0.141936E+01 0.000000E+01 0.000000E+01-0.853125E-01-0.118532E+00
0.625000E-03 0.145902E+01 0.000000E+01 0.000000E+01-0.846875E-01-0.117620E+00
0.250000E-02 0.148545E+01 0.187500E-02 0.149205E+01-0.803125E-01-0.111109E+00
0.937500E-02 0.153497E+01 0.281250E-02 0.154488E+01-0.681250E-01-0.923739E-01
  AERA 1     LEVER ARM      AREA 2        LEVER ARM   SUM FOR          SUM MOMENTS
0.316875E-01 0.194990E+00 0.487500E-02 0.259976E+00 0.365625E-01-0.744612E-02
0.231562E-01 0.584724E+00 0.914062E-02 0.519806E+00 0.688594E-01-0.257375E-01
0.175000E-01 0.978704E+00 0.312500E-02 0.912284E+00 0.894844E-01-0.457157E-01
0.487500E-02 0.124234E+01 0.406250E-03 0.122081E+01 0.947656E-01-0.522681E-01
0.162500E-02 0.143589E+01 0.406250E-02 0.139292E+01 0.100453E+00-0.602601E-01
STOP
```

APPENDIX X

DATA FILE AND OUTPUT

(Force and moment balance)

```
FO.WNME,RETR1
AS 18=*3
AS 12=REFILE14
VX.N

   AREA1     LEVER ARM     AREA 2       LEVER ARM    SUM FOR          SUM MOMENTS
0.150937E-01 0.104998E+00 0.131250E-02 0.699995E-01 0.164062E-01 0.167669E-02
0.890625E-02 0.304961E+00 0.237500E-02 0.273305E+00 0.276875E-01 0.504185E-02
0.225000E-02 0.444879E+00 0.984375E-03 0.429890E+00 0.309219E-01 0.646600E-02
0.875000E-03 0.513147E+00 0.000000E+01 0.000000E+01 0.317969E-01 0.691501E-02
0.250000E-02 0.626328E+00 0.000000E+01 0.000000E+01 0.292969E-01 0.534919E-02
0.625000E-02 0.721981E+00-0.175781E-02 0.742768E+00 0.212891E-01-0.468838E-03
0.820312E-02 0.836691E+00-0.820312E-03 0.854139E+00 0.122656E-01-0.803298E-02
0.843750E-02 0.933875E+00-0.154688E-02 0.948819E+00 0.228125E-02-0.173803E-01
0.140937E-01 0.103347E+01-0.464062E-02 0.105172E+01-0.164531E-01-0.368264E-01
0.191250E-01 0.113299E+01-0.984375E-03 0.114791E+01-0.365625E-01-0.596248E-01
0.233750E-01 0.123241E+01-0.120313E-02 0.121419E+01-0.611406E-01-0.898931E-01
0.135000E-01 0.133173E+01-0.281250E-02 0.131683E+01-0.774531E-01-0.111575E+00
0.337500E-02 0.139622E+01-0.131250E-02 0.138961E+01-0.821406E-01-0.118111E+00
0.843750E-03 0.142267E+01 0.000000E+01 0.000000E+01-0.829844E-01-0.119311E+00
0.150000E-02 0.145572E+01 0.000000E+01 0.000000E+01-0.814844E-01-0.117128E+00
0.400000E-02 0.148545E+01 0.187500E-02 0.149205E+01-0.756094E-01-0.108389E+00
0.116250E-01 0.153497E+01 0.262500E-02 0.154488E+01-0.613594E-01-0.864892E-01
  AERA 1     LEVER ARM     AREA 2       LEVER ARM    SUM FOR          SUM MOMENTS
0.320625E-01 0.189991E+00 0.356250E-02 0.253311E+00 0.356250E-01-0.699399E-02
0.183750E-01 0.484843E+00 0.164063E-02 0.449874E+00 0.556406E-01-0.166411E-01
0.118750E-01 0.684557E+00 0.237500E-02 0.652949E+00 0.698906E-01-0.263209E-01
0.690625E-02 0.864109E+00 0.185937E-02 0.835860E+00 0.786562E-01-0.338429E-01
0.934375E-02 0.106334E+01 0.359375E-03 0.110148E+01 0.883594E-01-0.441743E-01
0.365625E-02 0.137142E+01 0.670313E-02 0.130691E+01 0.987187E-01-0.579489E-01
STOP
```

```
ED REFILE14

1  18  1   11.0          0.3125
2   0.0
3   0.21
4   0.4
5   0.49
6   0.56
7   0.66
8   0.785
9   0.89
10   0.98
11   1.09
12   1.18
13   1.29
14   1.38
15   1.42
16   1.44
17   1.47
18   1.51
19   1.57
20   0.27
21   0.23
22   0.15
23   0.08
24   0.0
25  -0.16
26  -0.25
27  -0.3
28  -0.41
29  -0.68
30  -0.75
31  -0.68
32  -0.48
33  -0.27
34   0.0
35   0.32
36   0.62
37   0.9
38   7  1   11.0          0.3125
39   0.0
40   0.38
41   0.59
42   0.78
43   0.95
44   1.18
45   1.57
46   0.27
47   0.33
48   0.28
49   0.20
50   0.13
51   0.14
52   0.03
```

```
ED REFILE10

1  17  1   11.0          0.3125
2   0.0
3   0.23
4   0.39
5   0.51
6   0.56
7   -.65
8   0.785
9   0.89
10   0.98
11   1.07
12   1.18
13   1.29
14   1.38
15   1.43
16   1.45
17   1.52
18   1.57
19   0.23
20   0.20
21   0.13
22   0.11
23    0.0
24  -0.12
25  -0.24
26  -0.28
27  -0.40
28  -0.56
29  -0.72
30  -0.60
31  -0.40
32  -0.22
33   0.0
34   0.7
35   0.94
36   7  1   11.0          0.3125
37   0.0
38   0.39
39   0.59
40   0.78
41   1.17
42   1.33
43   1.57
44   0.23
45   0.29
46   0.24
47   0.16
48   0.12
49   0.08
50   0.01
```

```
FO.WNME,RETR1
AS 18=*3
AS 12=REFILE10
VX.N

   AREA1      LEVER ARM      AREA 2       LEVER ARM    SUM FOR          SUM MOMENTS
0.143750E-01 0.114998E+00 0.107812E-02 0.766660E-01 0.154531E-01 0.173575E-02
0.650000E-02 0.309959E+00 0.175000E-02 0.283302E+00 0.237031E-01 0.424626E-02
0.412500E-02 0.449874E+00 0.375000E-03 0.429890E+00 0.282031E-01 0.626320E-02
0.859375E-03 0.526465E+00 0.000000E+01 0.000000E+01 0.290625E-01 0.671563E-02
0.226875E-01-0.246646E+00 0.000000E+01 0.000000E+01 0.517500E-01 0.111985E-02
0.538125E-01 0.674996E-01-0.269062E-01 0.306627E+00-0.289688E-01-0.107626E-01
0.787500E-02 0.836691E+00-0.656250E-03 0.854139E+00-0.375000E-01-0.179121E-01
0.787500E-02 0.933875E+00-0.168750E-02 0.948819E+00-0.470625E-01-0.268675E-01
0.112500E-01 0.102352E+01-0.225000E-02 0.103845E+01-0.605625E-01-0.407186E-01
0.192500E-01 0.112304E+01-0.275000E-02 0.114128E+01-0.825625E-01-0.654756E-01
0.206250E-01 0.123241E+01-0.206250E-02 0.121419E+01-0.105250E+00-0.933983E-01
0.112500E-01 0.133173E+01-0.281250E-02 0.131683E+01-0.119313E+00-0.112084E+00
0.343750E-02 0.140118E+01-0.140625E-02 0.139292E+01-0.124156E+00-0.118859E+00
0.687500E-03 0.143259E+01 0.000000E+01 0.000000E+01-0.124844E+00-0.119844E+00
0.765625E-02 0.149205E+01 0.000000E+01 0.000000E+01-0.117188E+00-0.108421E+00
0.109375E-01 0.153993E+01 0.187500E-02 0.154818E+01-0.104375E+00-0.886748E-01
  AERA 1      LEVER ARM      AREA 2       LEVER ARM    SUM FOR          SUM MOMENTS
0.280312E-01 0.194990E+00 0.365625E-02 0.259976E+00 0.316875E-01-0.641634E-02
0.150000E-01 0.489838E+00 0.156250E-02 0.456535E+00 0.482500E-01-0.144773E-01
0.950000E-02 0.684557E+00 0.237500E-02 0.652949E+00 0.601250E-01-0.225313E-01
0.146250E-01 0.973724E+00 0.243750E-02 0.908962E+00 0.771875E-01-0.389876E-01
0.400000E-02 0.124731E+01 0.100000E-02 0.122081E+01 0.821875E-01-0.451977E-01
0.750000E-03 0.144580E+01 0.262500E-02 0.140614E+01 0.855625E-01-0.499731E-01
STOP
```

```
ED REFILE12

17  1   11.0          0.3125
 0.00
 0.21
 0.4
 0.49
 0.55
 0.67
 0.785
 0.89
 0.981
 1.09
 1.177
 1.29
 1.374
 1.43
 1.45
 1.52
 1.57
 0.15
 0.14
 0.09
 0.06
 0.0
-0.12
-0.17
-0.22
-0.31
-0.48
-0.55
-0.5
-0.32
 0.0
 0.12
 0.6
 0.75
 6  1   11.0          0.3125
 0.0
 0.38
 0.78
 1.18
 1.49
 1.57
 0.15
 0.2
 0.11
 0.08
 0.0
-0.01
```

```
FO.WNME,RETR1
AS 18=*3
AS 12=REFILE12
VX.N

   AREA1     LEVER ARM      AREA 2       LEVER ARM    SUM FOR          SUM MOMENTS
0.918750E-02 0.104998E+00 0.328125E-03 0.699995E-01 0.951562E-02 0.987641E-03
0.534375E-02 0.304961E+00 0.148437E-02 0.273305E+00 0.163437E-01 0.302296E-02
0.168750E-02 0.444879E+00 0.421875E-03 0.429890E+00 0.184531E-01 0.395506E-02
0.562500E-03 0.509817E+00 0.000000E+01 0.000000E+01 0.190156E-01 0.424183E-02
0.225000E-02 0.629656E+00 0.000000E+01 0.000000E+01 0.167656E-01 0.282510E-02
0.431250E-02 0.726970E+00-0.898437E-03 0.746093E+00 0.115547E-01-0.980272E-03
0.557812E-02 0.836691E+00-0.820312E-03 0.854139E+00 0.515625E-02-0.634810E-02
0.625625E-02 0.934373E+00-0.127969E-02 0.949484E+00-0.237969E-02-0.134088E-01
0.105594E-01 0.103397E+01-0.289531E-02 0.105206E+01-0.158344E-01-0.273729E-01
0.130500E-01 0.113150E+01-0.951562E-03 0.114592E+01-0.298359E-01-0.432294E-01
0.176562E-01 0.123092E+01-0.882812E-03 0.121220E+01-0.483750E-01-0.660329E-01
0.840000E-02 0.132875E+01-0.236250E-02 0.131485E+01-0.591375E-01-0.803007E-01
0.280000E-02 0.138895E+01 0.000000E+01 0.000000E+01-0.619375E-01-0.841897E-01
0.375000E-03 0.143920E+01 0.000000E+01 0.000000E+01-0.615625E-01-0.836500E-01
0.262500E-02 0.148049E+01 0.525000E-02 0.149205E+01-0.536875E-01-0.719305E-01
0.937500E-02 0.153993E+01 0.117188E-02 0.154818E+01-0.431406E-01-0.556794E-01
  AERA 1     LEVER ARM      AREA 2       LEVER ARM    SUM FOR          SUM MOMENTS
0.178125E-01 0.189991E+00 0.296875E-02 0.253311E+00 0.207812E-01-0.413622E-02
0.137500E-01 0.579731E+00 0.562500E-02 0.513147E+00 0.401562E-01-0.149940E-01
0.100000E-01 0.978704E+00 0.187500E-02 0.912284E+00 0.520312E-01-0.264916E-01
0.387500E-02 0.128042E+01 0.000000E+01 0.000000E+01 0.559062E-01-0.314532E-01
0.125000E-03 0.153827E+01 0.000000E+01 0.000000E+01 0.557812E-01-0.312609E-01
STOP
```

APPENDIX XI

CONVERSION OF RAW DATA TO INPUT FOR COMPUTER PROGRAM

This conversion deals with data preparation pertaining to the curve of initial residual stresses in the tube shown in Figure 3.28.

I) Definition of areas of forces limited by the curve and the X axis.

a) The curve of initial residual stresses obtained from measured strains in the tube with variable 4A and 4B, in Figure 3.28, has been drawn as follows:

-The longitudinal residual stresses computed from measured strains for 11 holes with hole depth of 0.125", shown in Table 3.3 (column 4) divided by yield stress, give ratios of σ_L/σ_y (column 6).

-The position of each hole is located by the ratio d/r, r being the radius of the steel tube (r = 11") and d, the distance in inch of the hole to the longitudinal weld, measured by cutting the tube with a plane passing by the weld and the tube center and flattening the cylinder on a plane parallel to the XY plane in Figure 4.8.

-On a system of perpendicular axes, plot d/r on the abscissa, σ_L/σ_y on the ordinate. A polygonal line (see Figure A.11.1) forms the curve which represents the pattern of distribution of longitudinal residual stresses at each of the 11 holes drilled in the tube.

-For obtaining a better curve which may reflect the continuity of the stresses present in the tube and give a closer value of the stress at any hole drilled on the semi-circular section of the 11 holes, a continuous smooth curved line was drawn, passing through the points of the above polygonal line (see Figure A.11.1).

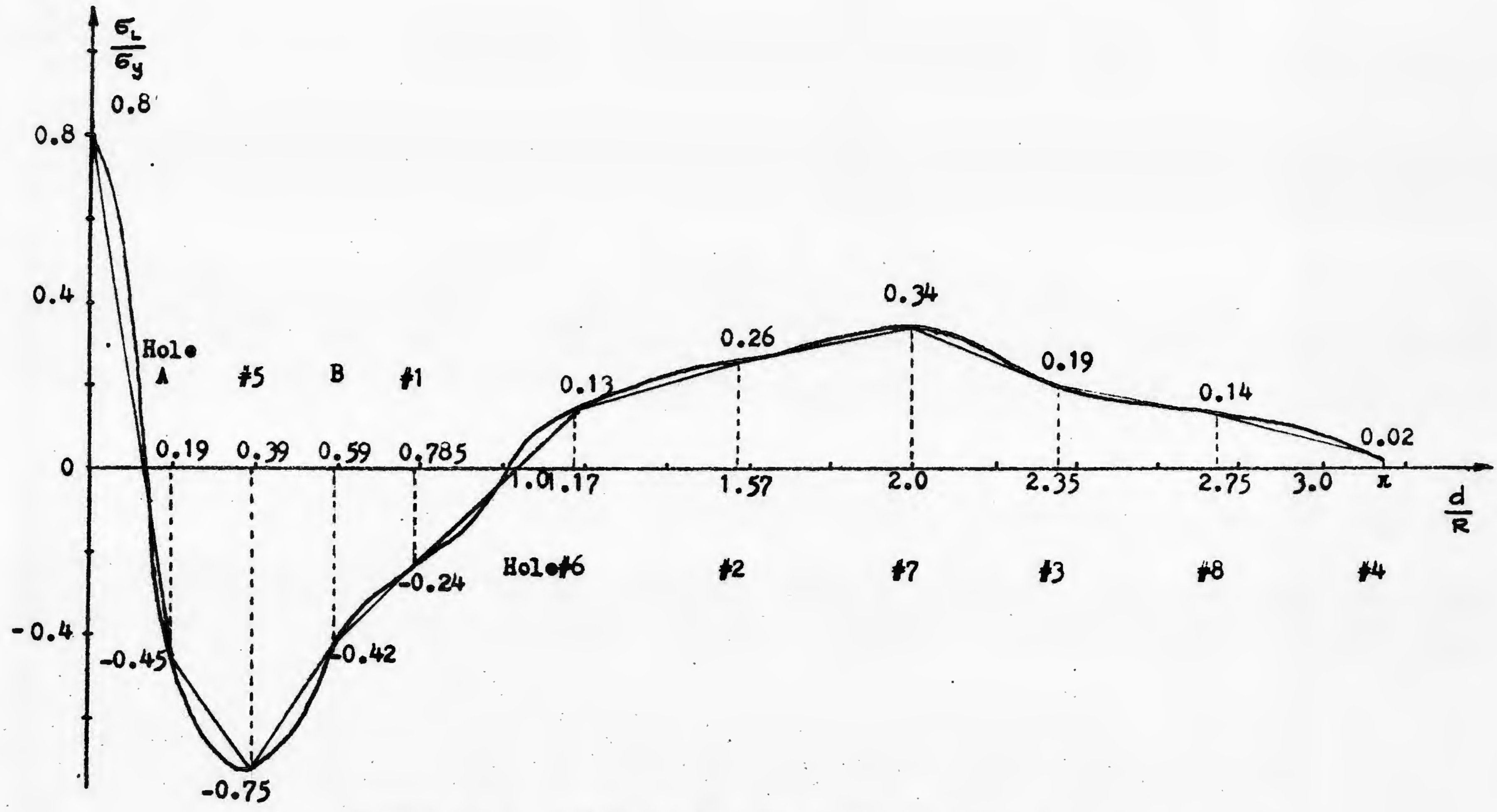


Figure A.11.1 Polygonal line and smooth continuous curve representing residual stresses, Table 3.11, case 1.

b) Taking the moment of an area of forces about an axis, is to compute the product of the area limited by the curve and the axis of abscissa and the lever arm, distance of the center of gravity of the area to the axis. As the above curve is not defined by an algebraic equation, the area limited by the curve and the axis of abscissa is calculated by a graphical method by approximation. The curve is approximated by a polygonal line (different from the above) which follows the curve as closely as possible, that is, when curvature is high, the inscribed line comprises short straight lines. The areas limited by the inscribed polygonal line is thus divided into trapezoids and triangles of which the evaluation of area is simple.

This principle is applied to the curve of residual stresses in Figure 3.28, and the polygonal line inscribed in the curve defines elementary areas of forces, shown in Figure A.11.2. As the points forming the trapezoids or triangles of forces are known by their coordinates, it is simple to calculate the areas of forces.

II) <u>Conversion of tube data to input for the computer program</u>.

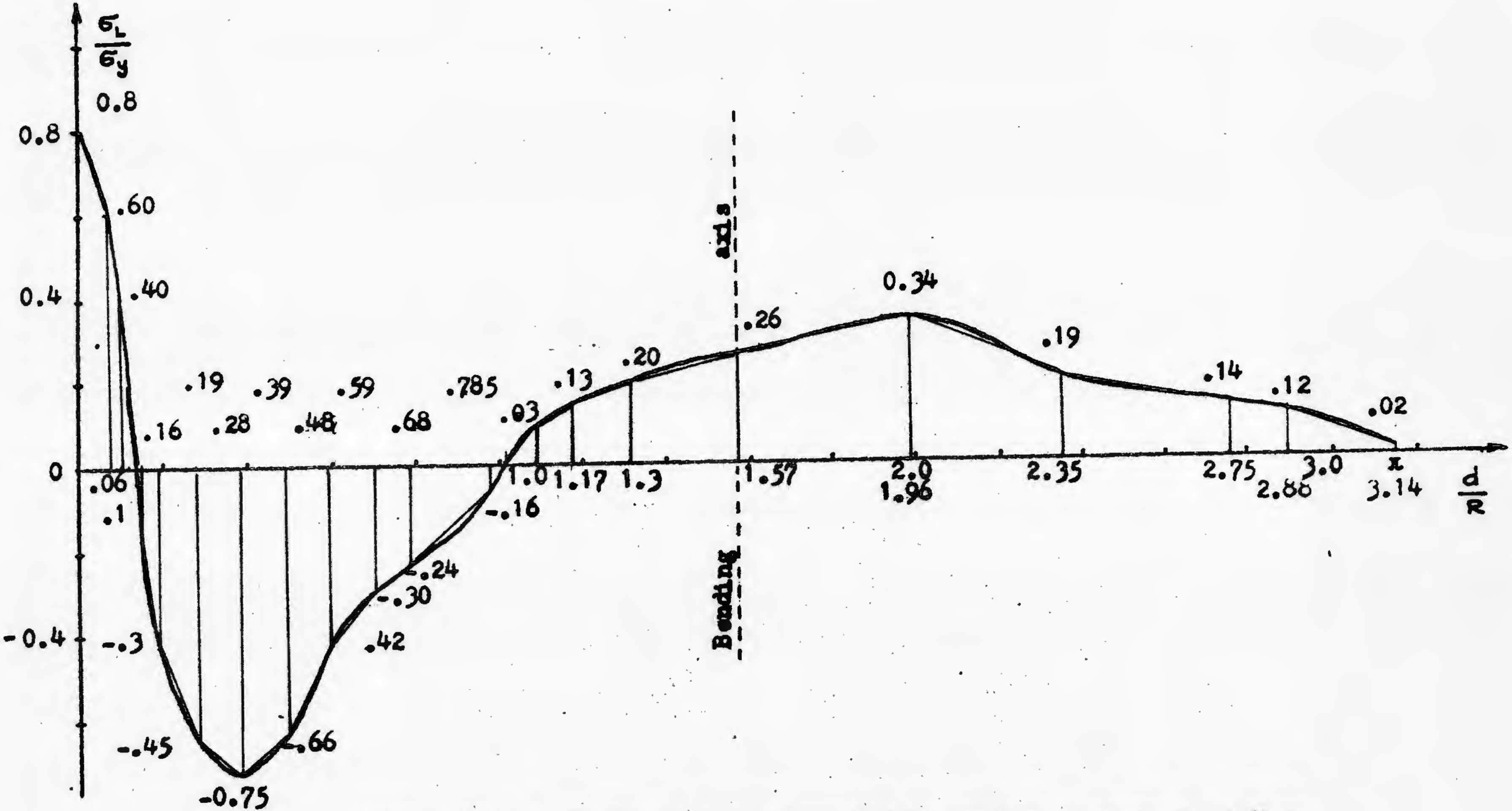
a) Summation of forces.

Tensions are represented by positive areas and compressions, by negative areas.

The computer program defines the summation of forces by adding algebraically positive and negative areas.

b) Summation of moments.

It has been pointed out in paragraph 4.1.1 that the sign of moments taken about the Z axis depends on the position of forces above or below the Z axis, therefore, the moment of forces is computed



<u>Figure A.11.2</u> **Polygonal line inscribed in continuous curve to define elementary areas of forces(Moment taken about bending axis, abscissa 1.57)**

separately for forces above the Z axis and for forces below this axis.

Sample calculation. (See Figure A.11.3)

A) For areas of forces below the Z axis, abscissa 1.57.

First trapezoid. (at right hand side of the bending axis)

Abscissa of first side, X_1 (new axis) = 0.0

Abscissa of second side $X_2 = 1.96 - 1.57 = 0.39$

Length of first side $Y_1 = 0.26$

Length of second side $Y_2 = 0.34$

Second trapezoid.

Abscissa of first side, $X_2 = 0.39$

Abscissa of second side $X_3 = 2.35 - 1.57 = 0.78$

Length of first side, $Y_2 = 0.34$

Length of second side $Y_3 = 0.19$

Third trapezoid.

Abscissa of first side, $X_3 = 0.78$

Abscissa of second side, $X_4 = 2.75 - 1.57 = 1.18$

Length of first side, $Y_3 = 0.19$

Length of second side, $Y_4 = 0.14$

Fourth trapezoid.

Abscissa of first side, $X_4 = 1.18$

Abscissa of second side $X_5 = 2.88 - 1.57 = 1.31$

Length of first side, $Y_4 = 0.14$

Length of second side, $X_5 = 0.12$

Fifth trapezoid.

Abscissa of first side, $X_5 = 1.31$

Abscissa of second side, $X_6 = 3.14 - 1.57 = 1.57$

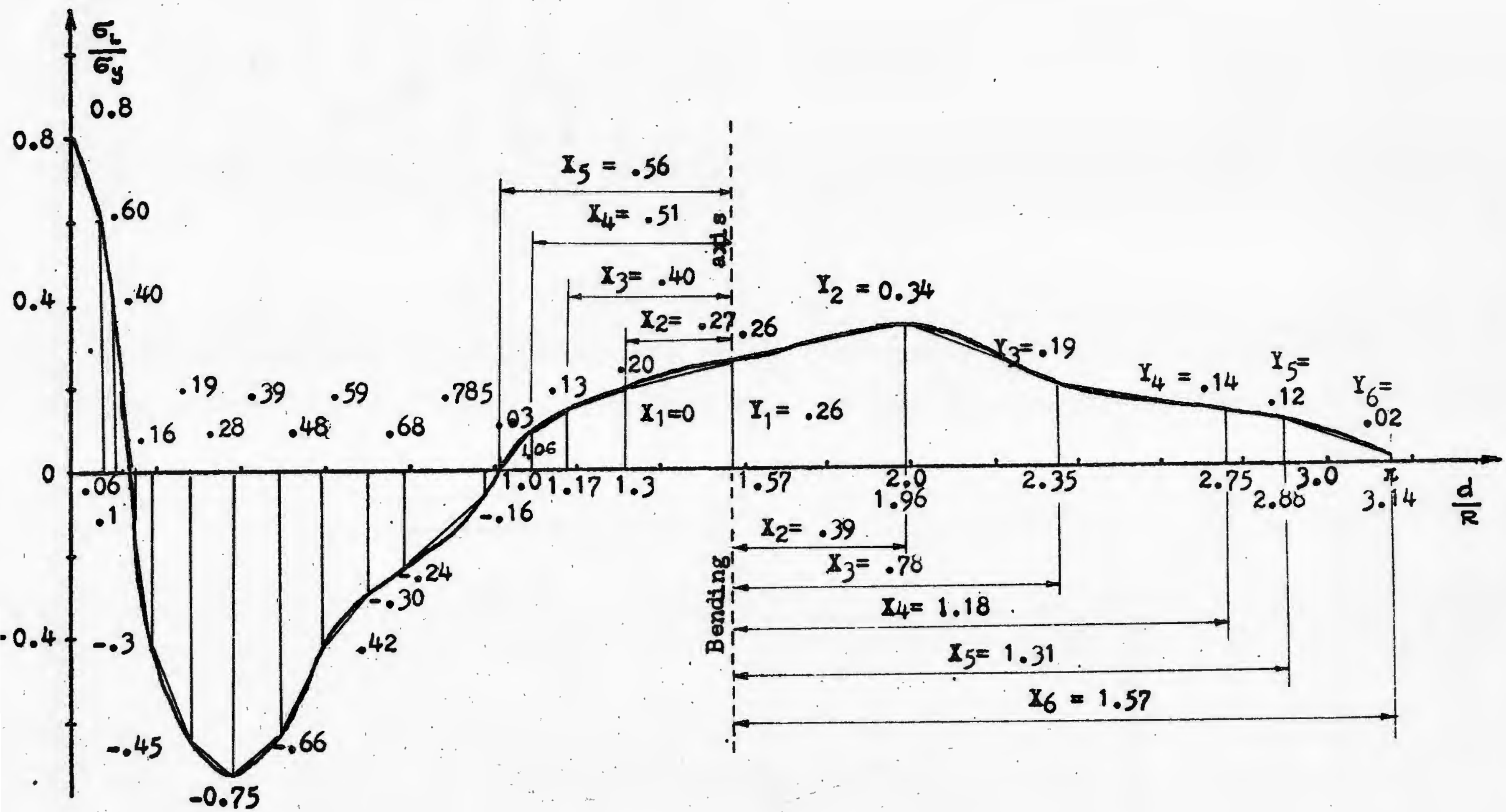


Figure A.11.3 Conversion of tube data to input for the computer program.

Length of first side, $Y_5 = 0.12$

Length of second side, $Y_6 = 0.02$

In Appendix IX, the input will be read as follows:

First reading. (Format statement #10)

I = number of data = 6 for this case.

J = number of set of data = 1

R = radius of the tube = 11.0 (inches)

T = thickness of the tube = 0.3125 (inches)

(see Appendix IX B, line 38)

Second reading. (Format statement #20)

X(N), by turns, equals

$X_1 = 0.0$, $X_1 = 0.34$, $X_3 = 0.78$, $X_4 = 1.18$, $X_5 = 1.31$, $X_6 = 1.57$

(see Appendix IX B. lines 38 to 44)

Third reading. (Format statement #40)

Y(N), by turns, equals

$Y_1 = 0.26$, $Y_2 = 0.34$, $Y_3 = 0.19$, $Y_4 = 0.14$, $Y_5 = 0.12$, $Y_6 = 0.02$

(see Appendix IX b, lines 45 to 50)

B) For areas of forces above the Z axis.

The same approach will lead to the input presented in Appendix IX B, line 1 to line 37.